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Unit 3: Probability

Lesson 3.4: Mutually exclusive and non-mutually exclusive Event

Investigate: A standard deck of playing cards is represented below.

A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣

Create a Venn diagram to represent the event of picking a club or spade.	Create a Venn diagram to represent the event of picking a diamond or face card.
Extend: Find the probability of randomly drawing either a club or a spade from a standard deck of cards.	Extend: Find the probability of randomly drawing either a diamond or a face card.
This is an example of two events are:	This is an example of two events are:

Mutually exclusive or disjoint events have different attributes and CAN NOT occur at the same time. For example, in a private school, a staff could be a principle or a cleaner, but not both. (rarely maybe sometimes☺)

Another easy example: A pair of dice is rolled. The events of rolling a 9 and of rolling a double have NO outcomes in common. These two events are mutually exclusive. BUT the events of rolling a 6 and of rolling a double have the outcome (3, 3) in common. These two events are **NON** mutually exclusive.

Hence **non-mutually** exclusive events have common characteristics which means they **CAN** occur simultaneously.

Addition Principle (Rule of Sum) for Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$

Principle of inclusion & Exclusion for Non-mutually exclusive events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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The difference between mutually exclusive and independent events:

- Events are mutually exclusive if the occurrence of one event excludes the occurrence of the other(s). Mutually exclusive events cannot happen at the same time.

  For example: when tossing a coin, the result can either be heads or tails but cannot be both. **(ADD)**

- Events are independent if the occurrence of one event does not influence (and is not influenced by) the occurrence of the other(s).

  For example: when tossing two coins, the result of one flip does not affect the result of the other. **(MULTIPLY)**

Example 1: Classify each pair of events as mutually exclusive or non-mutually exclusive.

1) Randomly selecting a student with brown eyes & Randomly selecting a student on the honor roll
2) Having an even number of students in your class & Having an odd number of students in your class
3) Getting an A on the next test & Passing the next test
4) Rolling a six with a die & Rolling a prime number with a die

Example 2: A certain provincial park has 220 campsites. A total of 80 sites have electricity. Of the 52 sites on the lakeshore, 22 of them have electricity. If a site is selected at random, what is the probability that:

a) it will be on the lakeshore?

b) it will either have electricity or be on the lakeshore? i.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

c) It will be on the lakeshore and not have electricity?

Let P (A) = lakeshore and P (B) = electricity

Communicate Your Understanding

1. Are an event and its complement mutually exclusive? Explain.
2. Explain how to determine the probability of randomly throwing either a composite number or an odd number using a pair of dice.
3. a) Explain the difference between independent events and mutually exclusive events.
   b) Support your explanation with an example of each.
   c) Why do you add probabilities in one case and multiply them in the other?

Practise

A

1. Classify each pair of events as mutually exclusive or non-mutually exclusive.

	Event A	Event B
a)	Randomly drawing a grey sock from a drawer	Randomly drawing a wool sock from a drawer
b)	Randomly selecting a student with brown eyes	Randomly selecting a student on the honour roll
c)	Having an even number of students in your class	Having an odd number of students in your class
d)	Rolling a six with a die	Rolling a prime number with a die
e)	Your birthday falling on a Saturday next year	Your birthday falling on a weekend next year
f)	Getting an A on the next test	Passing the next test
g)	Calm weather at noon tomorrow	Stormy weather at noon tomorrow
h)	Sunny weather next week	Rainy weather next week

2. Nine members of a baseball team are randomly assigned field positions. There are three outfielders, four infielders, a pitcher, and a catcher. Troy is happy to play any position except catcher or outfielder. Determine the probability that Troy will be assigned to play
   a) catcher
   b) outfielder
   c) a position he does not like

3. A car dealership analysed its customer database and discovered that in the last model year, 28% of its customers chose a 2-door model, 46% chose a 4-door model, 19% chose a minivan, and 7% chose a 4-by-4 vehicle. If a customer was selected randomly from this database, what is the probability that the customer
   a) bought a 4-by-4 vehicle?
   b) did not buy a minivan?
   c) bought a 2-door or a 4-door model?
   d) bought a minivan or a 4-by-4 vehicle?

Apply, Solve, Communicate

B

4. As a promotion, a resort has a draw for free family day-passes. The resort considers July, August, March, and December to be "vacation months."

a) If the free passes are randomly dated, what is the probability that a day-pass will be dated within

i) a vacation month?

ii) June, July, or August

b) Draw a Venn diagram of the events in part a).

5. A certain provincial park has 220 campsites. A total of 80 sites have electricity. Of the 52 sites on the lakeshore, 22 of them have electricity. If a site is selected at random, what is the probability that

a) it will be on the lakeshore?

b) it will have electricity?

c) it will either have electricity or be on the lakeshore?

d) it will be on the lakeshore and not have electricity?

6. A market-research firm monitored 1000 television viewers, consisting of 800 adults and 200 children, to evaluate a new comedy series that aired for the first time last week. Research indicated that 250 adults and 148 children viewed some or all of the program. If one of the 1000 viewers was selected, what is the probability that

a) the viewer was an adult who did not watch the new program?

b) the viewer was a child who watched the new program?

c) the viewer was an adult or someone who watched the new program?

7. **Application** In an animal-behaviour study, hamsters were tested with a number of intelligence tasks, as shown in the table below.

Number of Tests	Number of Hamsters
0	10
1	6
2	4
3	3
4 or more	5

If a hamster is randomly chosen from this study group, what is the likelihood that the hamster has participated in

a) exactly three tests?

b) fewer than two tests?

c) either one or two tests?

d) no tests or more than three tests?

8. **Communication**

a) Prove that, if A and B are non-mutually exclusive events, the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

b) What can you conclude if $P(A \text{ and } B) = 0$? Give reasons for your conclusion.

9. **Inquiry/Problem Solving** Design a game in which the probability of drawing a winning card from a standard deck is between 55% and 60%.

10. Chapter Problem: Determine the probability that a captured deer has either cross-hatched antlers or bald patches. Are these events mutually exclusive? Why or why not?

11. The eight members of the debating club pose for a yearbook photograph. If they line up randomly, what is the probability that

a) either Hania will be first in the row or Aaron will be last?

b) Hania will be first and Aaron will not be last?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application

12. Consider a Stanley Cup playoff series in which the Toronto Maple Leafs hockey team faces the Ottawa Senators. Toronto hosts the first, second, and if needed, fifth and seventh games in this best-of-seven contest. The Leafs have a 65% chance of beating the Senators at home in the first game. After that, they have a 60% chance of a win at home if they won the previous game, but a 70% chance if they are bouncing back from a loss. Similarly, the Leafs' chances of victory in Ottawa are 40% after a win and 45% after a loss.

a) Construct a tree diagram to illustrate all the possible outcomes of the first three games.

b) Consider the following events:

A = {Leafs lose the first game but go on to win the series in the fifth game}

B = {Leafs win the series in the fifth game}

C = {Leafs lose the series in the fifth game}

Identify all the outcomes that make up each event, using strings of letters, such as *LLSLL*. Are any pairs from these three events mutually exclusive?

c) What is the probability of event A in part b)?

d) What is the chance of the Leafs winning in exactly five games?

e) Explain how you found your answers to parts c) and d).

C

13. A grade 12 student is selected at random to sit on a university liaison committee. Of the 120 students enrolled in the grade 12 university-preparation mathematics courses,

- 28 are enrolled in data management only
- 40 are enrolled in calculus only
- 15 are enrolled in geometry only
- 16 are enrolled in both data management and calculus
- 12 are enrolled in both calculus and geometry
- 6 are enrolled in both geometry and data management
- 3 are enrolled in all three of data management, calculus, and geometry

a) Draw a Venn diagram to illustrate this situation.

b) Determine the probability that the student selected will be enrolled in either data management or calculus.

c) Determine the probability that the student selected will be enrolled in only one of the three courses.

14. **Application** For a particular species of cat, the odds against a kitten being born with either blue eyes or white spots are 3:1. If the probability of a kitten exhibiting only one of these traits is equal and the probability of exhibiting both traits is 10%, what are the odds in favour of a kitten having blue eyes?

15. **Communication**

a) A standard deck of cards is shuffled and three cards are selected. What is the probability that the third card is either a red face card or a king if the king of diamonds and the king of spades are selected as the first two cards?

b) Does this probability change if the first two cards selected are the queen of diamonds and the king of spades? Explain.