Class practice: Independent/Dependent and Odds **Date: May 24th, 2019**

1. (2f) Suppose that an urn contains 8 red balls and 4 white balls. We draw 2 balls from the urn without replacement. Suppose that the balls have different weights, with each red ball having weight r and each white ball having weight w. Suppose that the probability that a given ball in the urn is the next one selected is its weight divided by the sum of the weights of all balls currently in the urn. What is the probability that both balls are red?

$$P(R_1R_2) = \frac{8r}{8r + 4w}\,\frac{7r}{7r + 4w}$$

2. (2c) In the card game bridge, the 52 cards are dealt out equally to 4 players, called East, West, North, and South. If North and South have a total of 8 spaces among them, what are the odds that East has 3 of the remaining 5 spades?

$$\frac{\binom{5}{3}\binom{21}{10}}{\binom{26}{13}} \approx .339$$

3. (2h) An ordinary deck of 52 playing cards is randomly divided into 4 piles of 13 cards each. Compute the probability that each pile has exactly 1 ace.

$$P(E_1E_2E_3E_4) = P(E_1)P(E_2|E_1)P(E_3|E_1E_2)P(E_4|E_1E_2E_3)$$

Now,

$$P(E_1) = 1$$

since E_1 is the sample space S. Also,

$$P(E_2|E_1) = \frac{39}{51} \quad \frac{52-13}{52-1}$$

since the pile containing the ace of spades will receive 12 of the remaining 51 cards and

$$P(E_3|E_1E_2) = \frac{26}{50}$$

since the piles containing the aces of spades and hearts will receive 24 of the remaining 50 cards. Finally,

$$P(E_4|E_1E_2E_3) = \frac{13}{49}$$

Therefore, the probability that each pile has exactly 1 ace is

$$P(E_1E_2E_3E_4) = \frac{39 \cdot 26 \cdot 13}{51 \cdot 50 \cdot 49} \approx .105$$

4. (3n) A bin contains 3 different types of disposable flashlights. the probability that a type 1 flashlight will give over 100 hours of use is 0.7, with the corresponding probabilities for type 2 and type 3 flashlights being 0.4 and 0.3, respectively. Suppose that 20 percent of the flashlights in the bin are type 1, 30 percent are type 2, and 50 percent are type 3.
   a) What is the probability that a randomly chosen flashlight will give more than 100 hours of use?
   b) Given that a flashlight lasted over 100 hours, what is the conditional probability that it was a type j flashlight, j = 1, 2, 3?

$$\begin{aligned} P(A) &= P(A|F_1)P(F_1) + P(A|F_2)P(F_2) + P(A|F_3)P(F_3) \\ &= (.7)(.2) + (.4)(.3) + (.3)(.5) = .41 \end{aligned}$$

$$\begin{aligned} P(F_j|A) &= \frac{P(AF_j)}{P(A)} \\ &= \frac{P(A|F_j)P(F_j)}{.41} \end{aligned}$$

$$\begin{aligned} P(F_1|A) &= (.7)(.2)/.41 = 14/41 \\ P(F_2|A) &= (.4)(.3)/.41 = 12/41 \\ P(F_3|A) &= (.3)(.5)/.41 = 15/41 \end{aligned}$$

1. (3i) An urn contains two types A coins and one type B coin. When a type A coin is flipped, it comes up heads with probability ¼, whereas when a type B coin is flipped, it comes up heads with probability ¾. A coin is randomly chosen from the urn and flipped. Given that the flip landed on heads, what is the odds that it was a type A coin?

$$\begin{aligned}\frac{P(A|\text{heads})}{P(A^c|\text{heads})} &= \frac{P(A)}{P(B)}\frac{P(\text{heads}|A)}{P(\text{heads}|B)} \\ &= \frac{2/3}{1/3}\frac{1/4}{3/4} \\ &= 2/3\end{aligned}$$

2. (3l) Suppose that we have 3 cards that are identical in form, except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the odd against that the other side is colored black?

$$\begin{aligned}P(RB|R) &= \frac{P(RB \cap R)}{P(R)} \\ &= \frac{P(R|RB)P(RB)}{P(R|RR)P(RR) + P(R|RB)P(RB) + P(R|BB)P(BB)} \\ &= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{(1)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + 0\left(\frac{1}{3}\right)} = \frac{1}{3}\end{aligned}$$

3. (3a) An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a person who is not accident prone. If we assume that 30 percent of the population is accident prone,
   a) what is the probability that a new policyholder will have an accident within a year of purchasing a policy?
   b) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

$$P(A A_1) + P(A^c A_1)$$

$$\begin{aligned}P(A_1) &= P(A_1|A)P(A) + P(A_1|A^c)P(A^c) \\ &= (.4)(.3) + (.2)(.7) = .26\end{aligned}$$

$$\begin{aligned}P(A|A_1) &= \frac{P(AA_1)}{P(A_1)} \\ &= \frac{P(A)P(A_1|A)}{P(A_1)} \\ &= \frac{(.3)(.4)}{.26} = \frac{6}{13}\end{aligned}$$

4. (3f) At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, how certain of the guilt of the suspect should the inspector now be, if it turns out that the suspect has the characteristic?

Solution. Letting G denote the event that the suspect is guilty and C the event that he possesses the characteristic of the criminal, we have

$$\begin{aligned}P(G|C) &= \frac{P(GC)}{P(C)} \\ &= \frac{P(C|G)P(G)}{P(C|G)P(G) + P(C|G^c)P(G^c)} \\ &= \frac{1(.6)}{1(.6) + (.2)(.4)} \\ &\approx .882\end{aligned}$$