



**Unit 7: Normal distribution**

**Lesson 7.2: Confidence level and confidence intervals**

**Part I: Definitions**

**A. What's a Confidence Interval?**

Newspapers, magazines and other forms of media will often give statistics on an issue and provide the level of certainty or accuracy associated with the information.

**Recall:** Suppose we want to report on the mean of our **population**. We collect a **sample** so that we can **estimate** the mean of our **population**. If we conduct a survey that *is not* a census, we cannot truly determine the population mean,  $\mu$ ; instead, we can determine a **confidence interval** that highlights ranges of values within which  $\mu$  is likely to fall. Because the normal distribution is symmetrical, these intervals are centered around the sample mean,  $\bar{x}$ . (Note: The closer  $n$  gets to  $N$ , and the smaller the standard deviation, the more confident we can be about our mean!)

**Ex. 1:** The statement, "We can conclude with 95% confidence that millionaires donate between 12.6% and 17.4% of their income to charity." It is saying that:

There is a \_\_\_\_\_% probability that millionaires donate between \_\_\_\_\_ and \_\_\_\_\_ of their income to charity.

**Ex. 2:** Interpret the statement: "A study showed that 60% of first-year post secondary enrollees do not come directly from high school. These results are accurate  $\pm 5\%$ , 19 times out of 20".

**Part II: Confidence interval with given probability, p.**

**Margin of error:**

For statistic with probability p, the margin of error is:

| CONFIDENCE LEVEL | Z-SCORE, $z_{\frac{\alpha}{2}}$ |
|------------------|---------------------------------|
| 90 %             | 1.645                           |
| 95 %             | 1.960                           |
| 99 %             | 2.576                           |

Where z is z-score specifically depending on level of confidence. The table here gives a list of common confidence levels and their associated z-scores.



**Example 3:** Lake Ontario is one of the largest lakes in Canada. In 2009, 234 lake trout were caught out of a total catch of 911. In 2012, 141 lake trout were caught out of a catch of 689.

- Determine the percent of lake trout caught for each year.
- Determine the margin of error for each year. Use a 95% confidence level.
- Determine the confidence interval for each year.

**Example 4:** The first Major League Baseball franchise outside of the United States was the Montreal Expos, who played from 1969 to 2004. In 2005, the Expos moved to Washington, DC, and are now known as the Washington Nationals. A survey was conducted to determine whether a major league baseball team should come back to Montreal. Of the 1598 people surveyed, 69% were in favour of baseball coming back.

- Determine the margin of error for this survey at a confidence level 95%.
- For what range of percent can you be 95% confident that people would be in favour of baseball returning to Montreal?
- A second survey at a confidence level of 95% found that 56% were in favour, with a margin of error of 5.2%. approximately how many people were surveyed?



**Practice 1:** An opinion poll surveys 100 households who were watching television at 1 particular time. Of these, 75% were watching *Hockey Night in Canada*.

- a) Determine the confidence interval at a 99% confidence level.
- b) How would a news source state the results?

**Part III: Confidence interval with given sample mean and sample standard deviation**

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| CONFIDENCE INTERVAL FOR A NORMAL DISTRIBUTION<br>$\left( \bar{x} - z_{\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right), \bar{x} + z_{\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right) \right) \quad \text{or} \quad \bar{x} \pm z_{\frac{\alpha}{2}} \left( \frac{\sigma}{\sqrt{n}} \right)$ |
|--|

| CONFIDENCE LEVEL | Z-SCORE, $z_{\frac{\alpha}{2}}$ |
|------------------|---------------------------------|
| 90 %             | 1.645                           |
| 95 %             | 1.960                           |
| 99 %             | 2.576                           |

*Note: the greater the sample size, the smaller the margin of error. The smaller the margin of error, the greater the accuracy of the measurement.*

**Example 5:** At the agricultural fair, the masses of 8 giant pumpkins entered in a contest were 11kg, 13kg, 15kg, 18kg, 12kg, 14kg, 10kg, 16kg. Results from past fairs suggest that the masses are normally distributed with a mean of 14.2kg and a standard deviation of 2.5kg. Determine a 90% confidence interval for the sample mean.



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**Practice 2:** A consumers' group tested batches of light bulbs to see how long they lasted. The results, in hours, from one batch were 998, 1234, 1523, 1760, 937, 1193, 996, 1002, 986, 1285, 1163, and 1716. The manufacturer claims that the life of the light bulbs is normally distributed with a mean of 1200 hours and a standard deviation of 420h. Determine the 99% confidence interval for the sample mean.