Unit 2 – Polynomials
Chapter 3.1: Polynomial functions

Consider the familiar functions:

Linear:

Quadratic:

We can continue this pattern:

Cubic:

Quartic:

Quintic:

In general, a polynomial function in standard form is: $\boldsymbol{f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}$
$\boldsymbol{where\ \{a_0, a_1, \ldots, a_{n-1}, a_n \in R\} and\ \{n \in W\}}$

Notes:
1) $\boldsymbol{a_n}$ is the leading coefficient
2) The degree of a polynomial is the value of the highest exponent
3) A polynomial in standard form has descending powers of x.

These are polynomial expressions.	These are not polynomial expressions.
$3x^2 - 5x + 3$	$\sqrt{x} + 5x^3$
$-4x + 5x^7 - 3x^4 + 2$	$\dfrac{1}{2x + 5}$
$\frac{2}{5}x^3 - 3x^5 + 4$	$6x^3 + 5x^2 - 3x + 2 + 4x^{-1}$
$\sqrt{4}x^3 - \frac{\sqrt{5}}{3}x^2 + 2x - \frac{1}{4}$	$\dfrac{3x^2 + 5x - 1}{2x^2 + x - 3}$
$3x - 5$	$4^x + 5$
-7	$\sin(x - 30)$
$-4x$	$x^2y + 3x - 4y^{-2}$
$(2x - 3)(x + 1)^2$	$3x^3 + 4x^{2.5}$

Recall: **Finite differences**

1) First differences are a constant for _______________.
2) Second differences are a constant for _________________.

Higher-order finite differences can be used to identify other polynomials from data points.

For an Order-N polynomial, the Nth difference will be a constant.

3rd degree

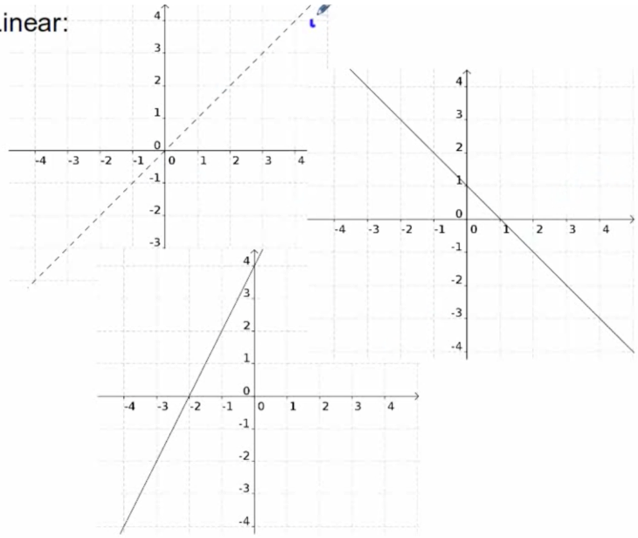
$y = 0.1x^3 - x^2 + 3x - 5$

x	y	D_1	D_2	D_3
−5	−57.5			
		52.5		
0	−5		−50	
		2.5		75
5	−2.5		25	
		27.5		75
10	25		100	
		127.5		75
15	152.5		175	
		302.5		
20	455			

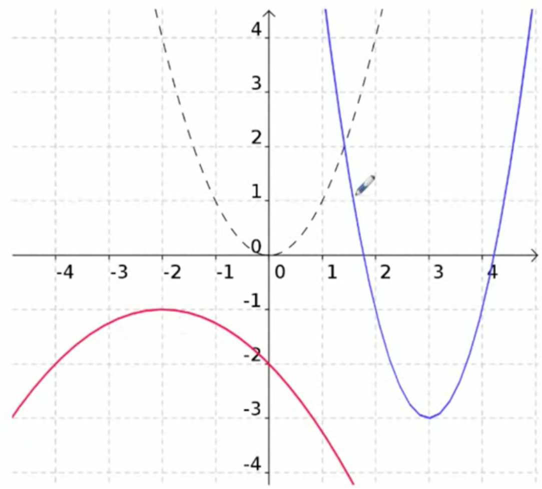
Domain is always $\{x|x \in R\}$.

Range varies according to graph (Parent + transformations).

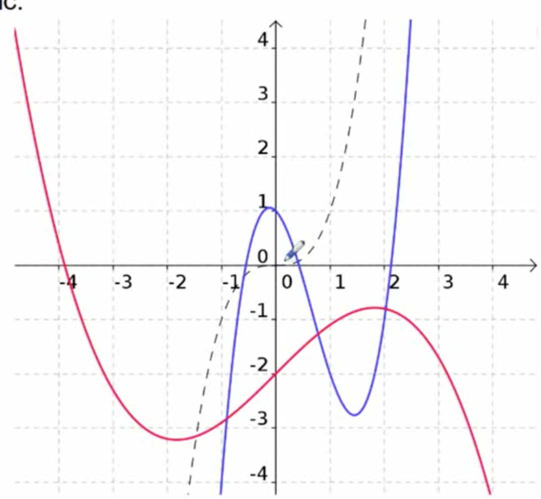
Linear:



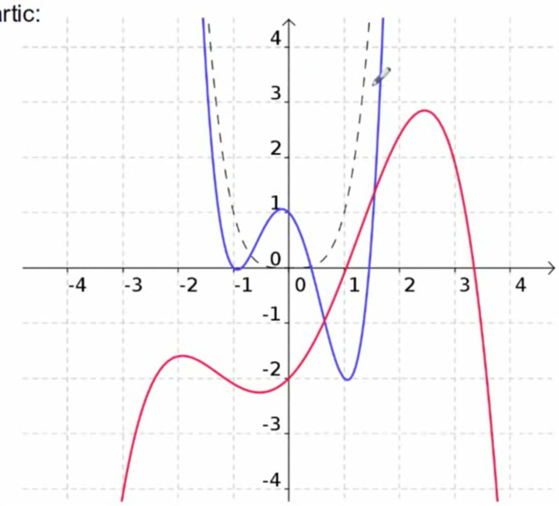
Quadratic:



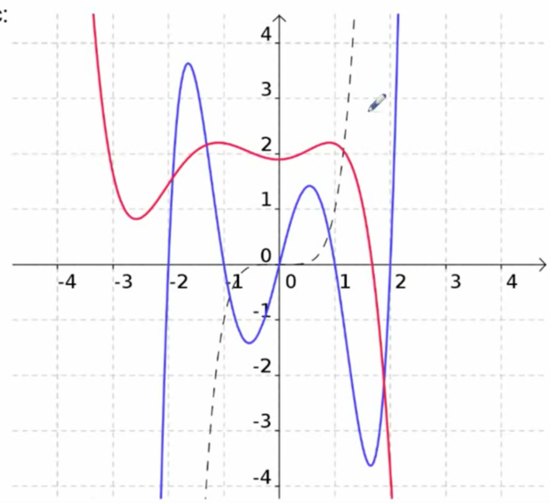
Cubic:



Quartic:



Quintic:



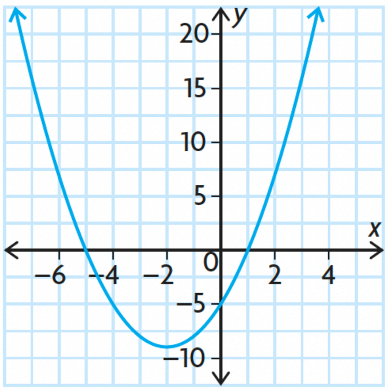
Suggested problems from textbook: pg 127 - 128/ #1, 2, 5

Investigation:
Chapter 3.2 → Characteristics of polynomial functions

How can you predict some of the characteristics of a polynomial function from its equation?

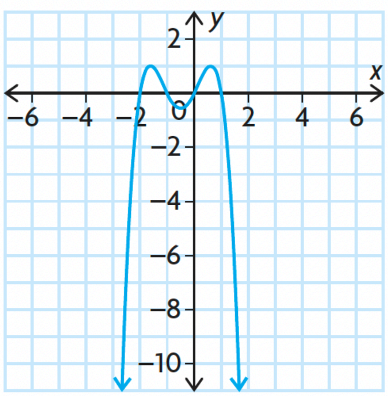
The graphs of some polynomial functions are shown below.

a)



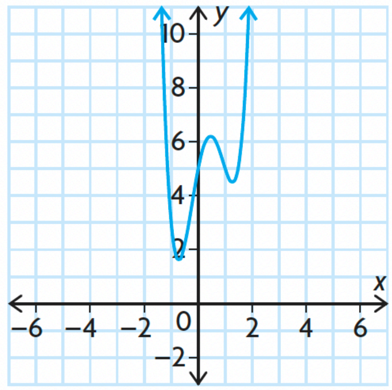
$f(x) = x^2 + 4x - 5$

b)



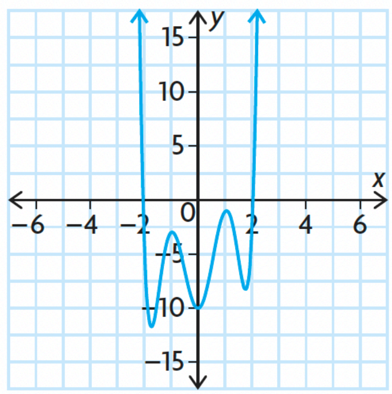
$f(x) = -x^4 - 2x^3 + x^2 + 2x$

c)



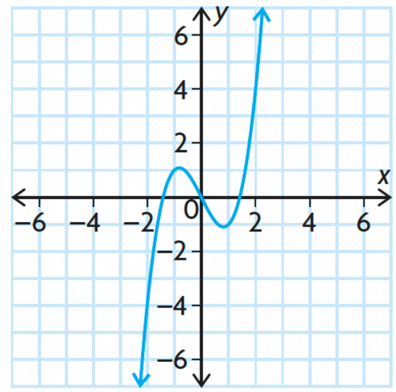
$f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$

d)



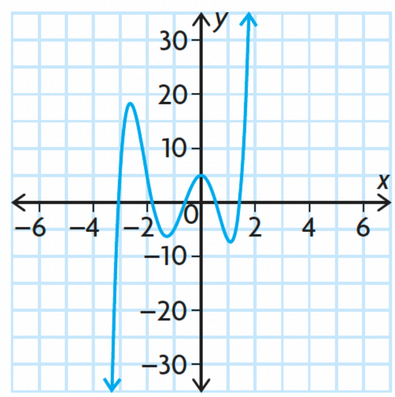
$P(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$

e)



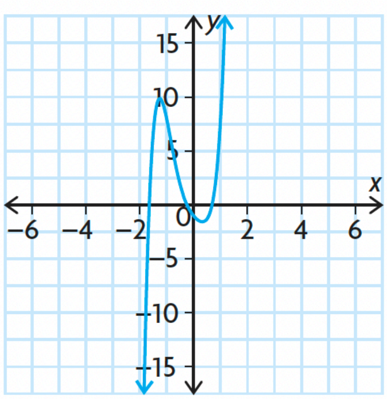
$f(x) = x^3 - 2x$

f)



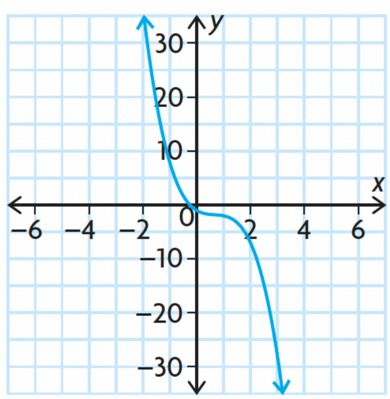
$f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$

g)



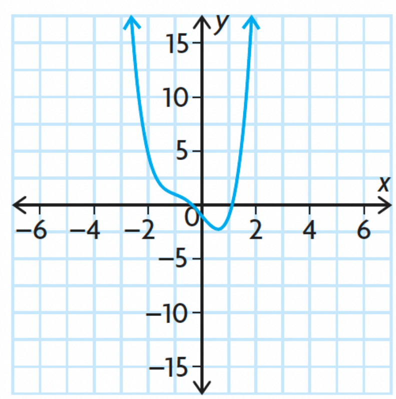
$f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$

h)



$f(x) = -2x^3 + 4x^2 - 3x - 1$

i)



$f(x) = x^4 + 2x^3 - 3x - 1$

Part A: In the following table, complete it using the equations and graphs given above.

Equation & Graph	Degree	Even or Odd degree?	Leading coefficient	End behaviors		Number of Turning points
				$x \to -\infty$	$x \to \infty$	
a)	2	even	+1	$y \to +\infty$	$y \to +\infty$	1
b)						
c)						
d)						
e)						
f)						
g)						
h)						
i)						

Part B: Create two new polynomial functions of degree greater than 2, one of even degree and one of odd degree. Do these new polynomial functions support your observation in part A?

Function 1:

Function 2:

Part C: What do you think is the maximum number of turning points that a polynomial function of degree *n* can have?

Part D: Graph the following functions using DESMOS (https://www.desmos.com/calculator).
Copy each graph roughly and its equation into the appropriate column of a table like the one shown below.

i) $f(x) = x^4 - 2x^2 + 1$

ii) $f(x) = x^3 + 3x^2 - 2x - 5$

iii) $f(x) = \frac{1}{2}x^{10} - \frac{1}{3}x^4 + x^2$

iv) $f(x) = x^3 + x$

v) $f(x) = -2x^6 + 3x^4$

vi) $f(x) = x^5 - 3x$

vii) $f(x) = x^2 - 3x + 4$

viii) $f(x) = 2x^7 - 3x^3 + 2x$

ix) $f(x) = -3x^4 + 2x^3 - 3x + 1$

x) $f(x) = x^2 - x$

Even Functions	Odd functions	Neither Even nor Odd functions
(symmetry in the y-axis)	(rotational symmetry around the origin)	(neither of these symmetries)

Part E: Is every function of even degree an even function?
Is every function of odd degree an odd function?
Hint: You could determine that by looking at graphs or algebraically.
i.e., Even function has $f(-x) = f(x)$; Odd function has $f(-x) = -f(x)$.

Part F: How can you use the equation of a polynomial function to describe its end behaviors, number of turning points, and symmetry?

Reflecting:
Part G: Why must all polynomial functions of even degree have an absolute maximum or absolute minimum?

Part H: Why must all polynomial functions of odd degree have at least one zero?

The end of investigation.