**Unit 2: Polynomial Functions**
**Chapter 3.4: Transformation**

Determining Transformed functions from graphs and equations:

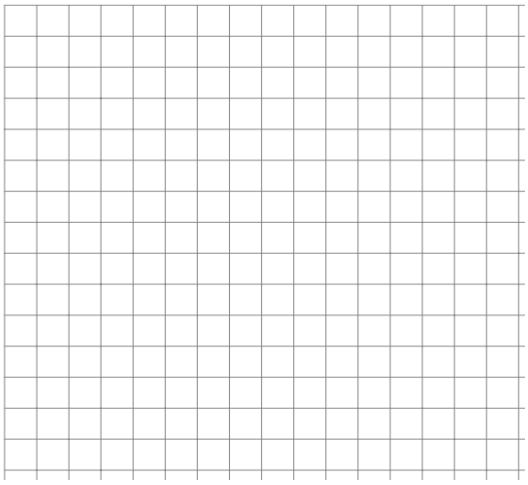
$$\boldsymbol{y = af[k(x - d)] + c}$$

For any single point, the transformations can be summarized as mapping notation:

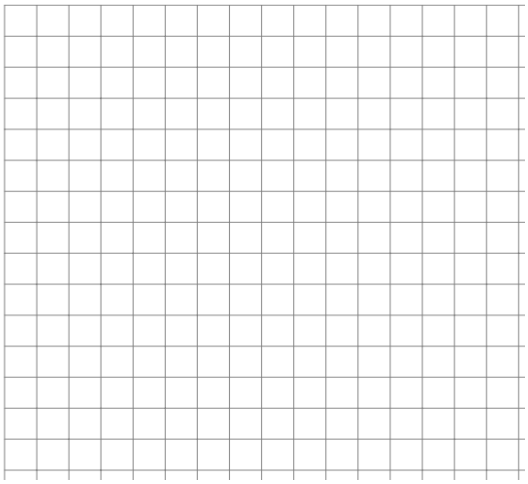
$$\boldsymbol{(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)}$$

Example 1: Reasoning about the order of transformations

Write mapping notation, and describe the steps of transformations that must be applied to $y = x^3$ (parent function) to graph $y = -8(\frac{1}{2}x + 1)^3 - 3$, and then graph both parent function and this newly transformed function.



Practice:
Write mapping notation, and describe the steps of transformations that must be applied to $y = x^4$ (parent function) to graph $y = -3[-2(x - 1)]^4 + 1$, and then graph both parent function and this newly transformed function.



## In Summary

### Key Ideas

- The polynomial function $y = a(k(x - d))^n + c$ can be graphed by applying transformations to the graph of the parent function $y = x^n$, where $n \in \mathbf{N}$. Each point $(x, y)$ on the graph of the parent function changes to $\left(\frac{x}{k} + d, ay + c\right)$.
- When using transformations to graph a function in the fewest steps, you can apply $a$ and $k$ together, and then $c$ and $d$ together.

### Need to Know

- In $y = a(k(x - d))^n + c$,
  - the value of $a$ represents a vertical stretch/compression and possibly a vertical reflection
  - the value of $k$ represents a horizontal stretch/compression and possibly a horizontal reflection
  - the value of $d$ represents a horizontal translation
  - the value of $c$ represents a vertical translation

## In Summary

### Key Ideas

- Transformations on a function $y = af(k(x - d)) + c$ must be performed in a particular order: horizontal and vertical stretches/compressions (including any reflections) must be performed before translations. All points on the graph of the parent function $y = f(x)$ are changed as follows: $(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$
- When using transformations to graph, you can apply $a$ and $k$ together, and then $c$ and $d$ together, to get the desired graph in the fewest number of steps.

### Need to Know

- The value of $a$ determines whether there is a vertical stretch or compression, or a reflection in the $x$-axis:
  - When $|a| > 1$, the graph of $y = f(x)$ is stretched vertically by the factor $|a|$.
  - When $0 < |a| < 1$, the graph is compressed vertically by the factor $|a|$.
  - When $a < 0$, the graph is also reflected in the $x$-axis.
- The value of $k$ determines whether there is a horizontal stretch or compression, or a reflection in the $y$-axis:
  - When $|k| > 1$, the graph is compressed horizontally by the factor $\frac{1}{|k|}$.
  - When $0 < |k| < 1$, the graph is stretched horizontally by the factor $\frac{1}{|k|}$.
  - When $k < 0$, the graph is also reflected in the $y$-axis.
- The value of $d$ determines whether there is a horizontal translation:
  - For $d > 0$, the graph is translated to the right.
  - For $d < 0$, the graph is translated to the left.
- The value of $c$ determines whether there is a vertical translation:
  - For $c > 0$, the graph is translated up.
  - For $c < 0$, the graph is translated down.