Unit 2 – Solving equations and inequalities

Chapter 4.1: Solving polynomial equations and word problems

Recall: To solve an equation means finding the real roots of the equation.

When solving a quadratic equation, there are several options, such as:

a) Factoring to find the zeros
b) Quadratic formula

Polynomial equations of degree 3 or higher can be solved by:

a) Graphing
b) Factoring down to degree 2 using factor theorem, long division or synthetic division, then applying one of the techniques listed above to fully factor.

Example 1: Solve $3x^3 + 8x^2 = -3x + 2$

Example 2: Determine the exact roots of $x^3 - 4x^2 + 2x + 3$

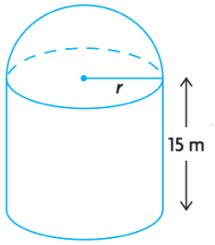
Step 1: rewrite the equation of it to equal to zero.
Step 2: define the resulting polynomial as a function and apply the factor theorem.
Step 3: factor out the first term, and repeat until in a fully factored form.
Step 4: find the roots of the equation (i.e., set it back to zero and solve)
Step 5: ignore solutions that are outside of the domain defined by the conditions of the problem.

Example 3: A box is in the shape of a rectangular prism. One side is a square, and the length is 12 units longer than the square sides. The volume of the box is 135 cubic units. What are the dimensions of the box?

Practice 1 :

Amelia's family is planning to build another silo for grain storage, identical to those they have on their farm. The cylindrical portion of those they currently have is 15 m tall, and the silo's total volume is 684π m^3.

Determine possible values for the radius of the silo.



Independent practice 2:

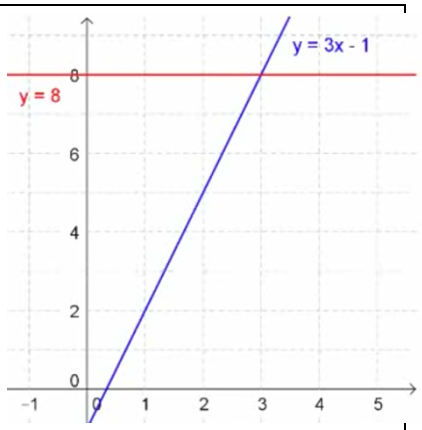
The paths of two orcas playing in the ocean were recorded by some oceanographers. The first orca's path could be modelled by the equation $h(t) = 2t^4 - 17t^3 + 27t^2 - 252t + 232$, and the second by $h(t) = 20t^3 - 200t^2 + 300t - 200$, where h is their height above/below the water's surface in centimetres and t is the time during the first 8 s of play. Over this 8-second period, at what times were the two orcas at the same height or depth?

Suggested questions from Textbook: pg 204/ #6def, 9, 10, 11, 15, 16

Unit 3 – Solving equations and inequalities
Chapter 4.2 – 4.3: Solving linear inequalities and polynomial inequalities

Part I: Linear inequalities

To solve an inequality, find all values that satisfy the inequality.



Consider $3x - 1 < 8$, the simplest way to visualize the solution is to graph and compare the LS and RS:

Where is the line y = 3x – 1 less than the line y = 8?

Or algebraically solve it:

Note: When multiplying or dividing by a negative value, the direction of the inequality must be switched.

For a double-inequality, perform each operation on all parts simultaneously and individually.

Example 1: Solve $10 \leq -3(2x - 5) - (3x - 7) < 25$.

Express your solution using:

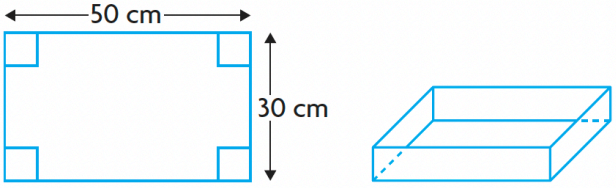
a) set notation
b) interval notation
c) a number line

Part II: Non - Linear Polynomial inequalities

Example 2: Solve $x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$ (Hint: factoring by grouping)

Example 3:

An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of such a box. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than 4000 cm^3.



Independent practice:

The elevation of a hiking trail is modelled by the function $h(x) = 2x^3 + 3x^2 - 17x + 12$, where h is the height measured in metres above sea level and x is the horizontal position from a ranger station measured in kilometres. If x is negative, the position is to the west of the station, and if x is positive, the position is to the east. Since the trail extends 4.2 km to the west of the ranger station and 4 km to the east, the model is accurate where $x \in [-4.2, 4]$.

How can we determine which sections of the trail are above the sea level?

Suggested questions from Textbook: pg 224 – 226. #2, 6de, 8, 10, 14; pg241. #10, 11