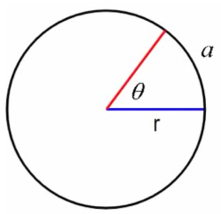
Unit 4 – Trigonometry

Chapter 6.1 – 6.3: Radian measure for angles and Radian angles in Cartesian plane

Part I: Radian measure and Angular Speed

Radian Measure



An angle measurement can be defined as the ratio of the arc length to the radius of a circle:

$$\theta = \frac{a}{r}$$

For a full circle, the arc length is the circumference: $C = 2\pi r$,

Therefore, the angle described by a full circle is: $360° = \frac{2\pi r}{r} = 2\pi = 1\,rotation$

The way to inter-change radian and degree:

$\because 2\pi\,radian = 360°$

$\therefore 1\,radian = \frac{360°}{2\pi} = \frac{180°}{\pi}$, whereas $1\,degree = \frac{2\pi}{360°} = \frac{\pi}{180°}$

Example 1: convert each of the following angles

a) $20°$

b) $225°$

c) $\frac{5\pi}{6}$

d) 1.75 radian

Example 2:

The London Eye Ferris wheel has a diameter of 135 m and completes one revolution in 30 min.

a) Determine the angular velocity, ω, in radians per second.

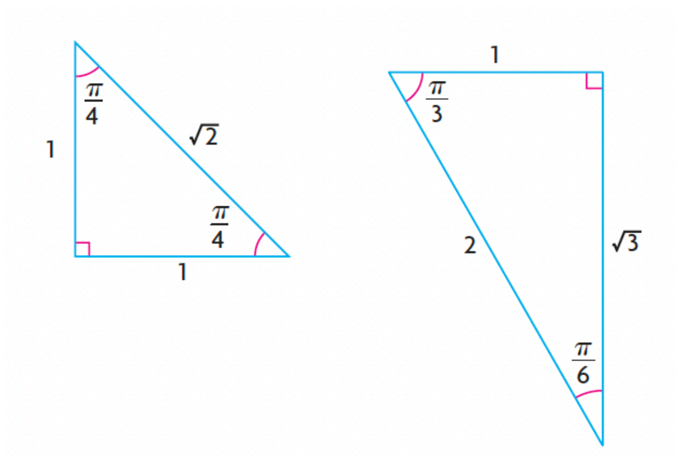
b) How far has a rider travelled at 10 min into the ride?

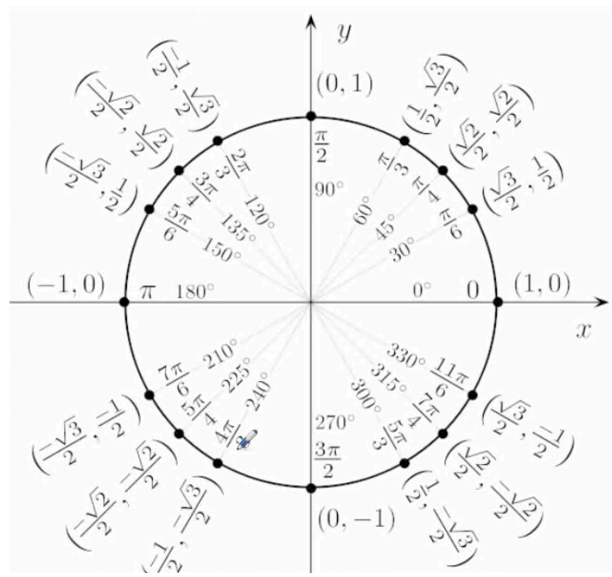
Part II: Radian angles in Cartesian plane

- Recall: For any angle of interest (θ), there are three primary trigonometric ratios.

$$sine\ of\ \theta = \frac{opposite}{hypotenuse}, \qquad cosine\ of\ \theta = \frac{adjacent}{hypotenuse}, \qquad tangent\ of\ \theta = \frac{opposite}{adjacent}$$

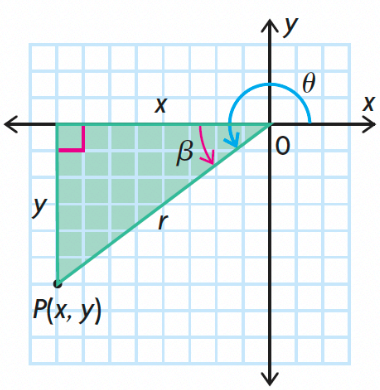
- Recall Special Triangles (angles in degrees and radians):





Need to Know

- The trigonometric ratios for any principal angle, θ, in standard position can be determined by finding the related acute angle, β, using coordinates of any point that lies on the terminal arm of the angle.

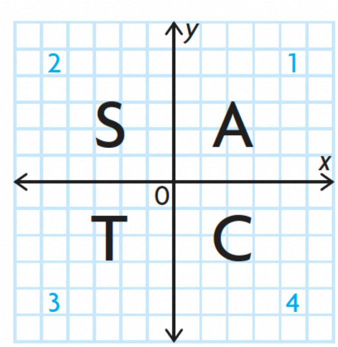


From the Pythagorean theorem, $r^2 = x^2 + y^2$, if $r > 0$.

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

- The CAST rule is an easy way to remember which primary trigonometric ratios are positive in which quadrant. Since r is always positive, the sign of each primary ratio depends on the signs of the coordinates of the point.
  - In quadrant 1, **A**ll (A) ratios are positive because both x and y are positive.
  - In quadrant 2, only **S**ine (S) is positive, since x is negative and y is positive.
  - In quadrant 3, only **T**angent (T) is positive because both x and y are negative.
  - In quadrant 4, only **C**osine (C) is positive, since x is positive and y is negative.



Example 2: Evaluate。

a) $sin\frac{5\pi}{3}$

b) $cos\frac{5\pi}{4}$

Example 3: Solve $tan\theta = -\frac{7}{24}$ for θ is between 0 and 2 π.