Unit 4 – Trigonometry

Chapter 6.3 – 6.4: Transformation of Trigonometric functions

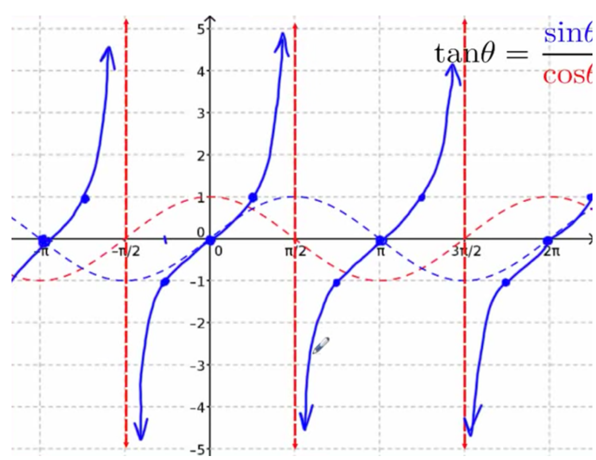
Graphing from key points

For sine and cosine, use points from the x- and y-axes on the unit circle.

$\theta \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$

For tangent, use a cycle between two vertical asymptotes:

$\theta \in \{-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}\}$



Transform each point $(x, y) \rightarrow (\frac{x}{k} + d, ay + c)$

Summary:

Characteristics	Sine	Cosine	Tangent
Domain			
Range			
Zeros			
Asymptotes			
Period			

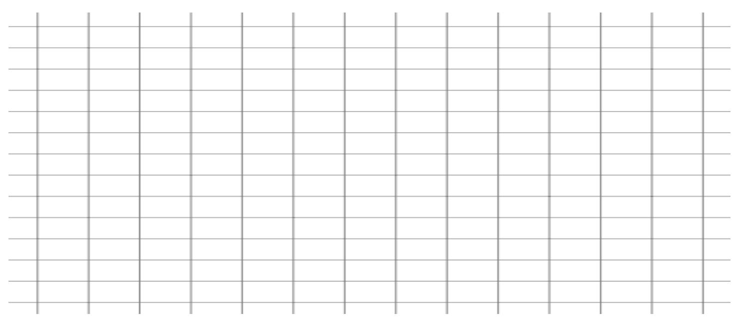
Graphing from key properties

a: vertical reflection and amplitude
k: horizontal reflection and period

sine and cosine: period $= \frac{2\pi}{k}$
tangent: period $= \frac{\pi}{k}$

d: phase shift of starting point
c: axis of the curve

Example 1:
A mass on a spring is pulled toward the floor and released, causing it to move up and down. Its height, in centimetres, above the floor after t seconds is given by the function $h(t) = 10 \sin(2\pi t + 1.5\pi) + 15, 0 \leq t \leq 3$. Sketch a graph of height versus time. Then use your graph to predict when the mass will be 18 cm above the floor as it travels in an upward direction.



Need to Know

- The parameters in the equations $f(x) = a \sin(k(x - d)) + c$ and $f(x) = a \cos(k(x - d)) + c$ give useful information about transformations and characteristics of the function.

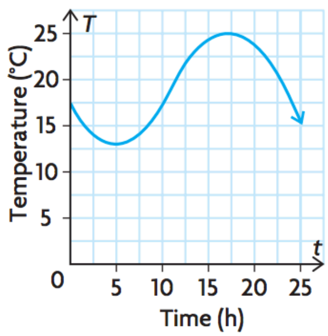
Transformations of the Parent Function	Characteristics of the Transformed Function
$\lvert a \rvert$ gives the vertical stretch/compression factor. If $a < 0$, there is also a reflection in the x-axis.	$\lvert a \rvert$ gives the amplitude.
$\left\lvert \frac{1}{k} \right\rvert$ gives the horizontal stretch/compression factor. If $k < 0$, there is also a reflection in the y-axis.	$\frac{2\pi}{\lvert k \rvert}$ gives the period.
d gives the horizontal translation.	d gives the horizontal translation.
c gives the vertical translation.	$y = c$ gives the equation of the axis.

- If the independent variable has a coefficient other than +1, the argument must be factored to separate the values of k and c. For example,

  $y = 3\cos(2x + \pi)$ should be changed to $y = 3\cos\left(2\left(x + \frac{\pi}{2}\right)\right)$.

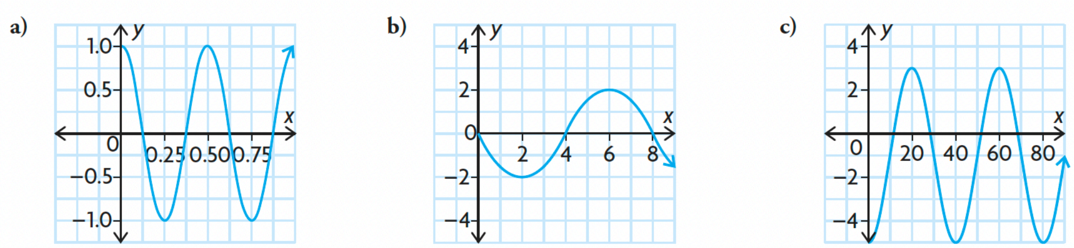
Example 2:

The following graph shows the temperature in Nellie's dorm room over a 24 h period.



Determine the equation of this sinusoidal function.

14. Determine a sinusodial equation for each of the following graphs.



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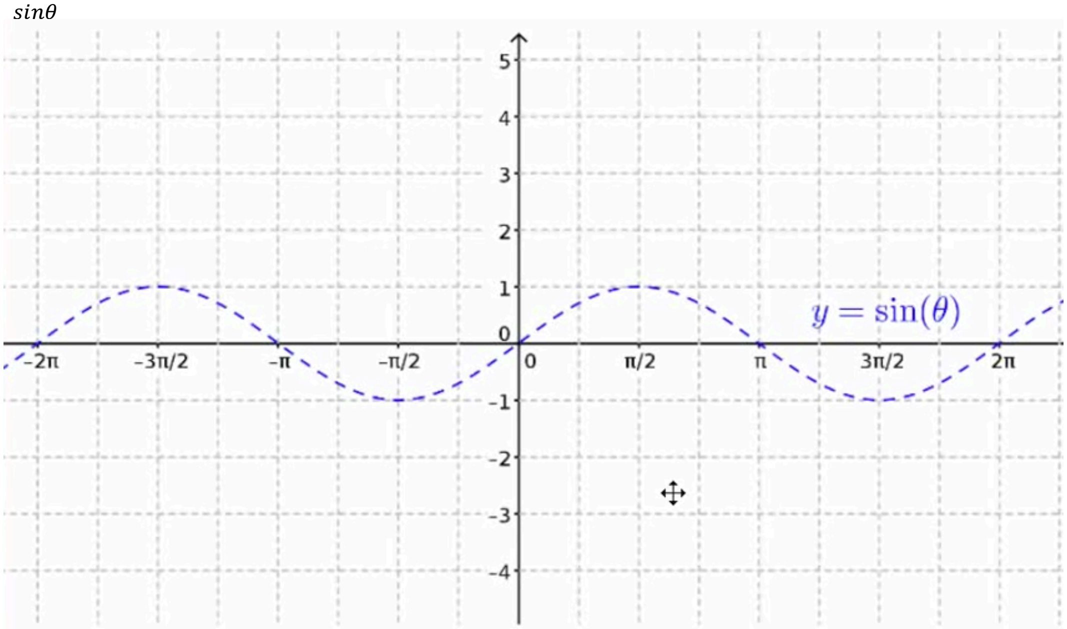
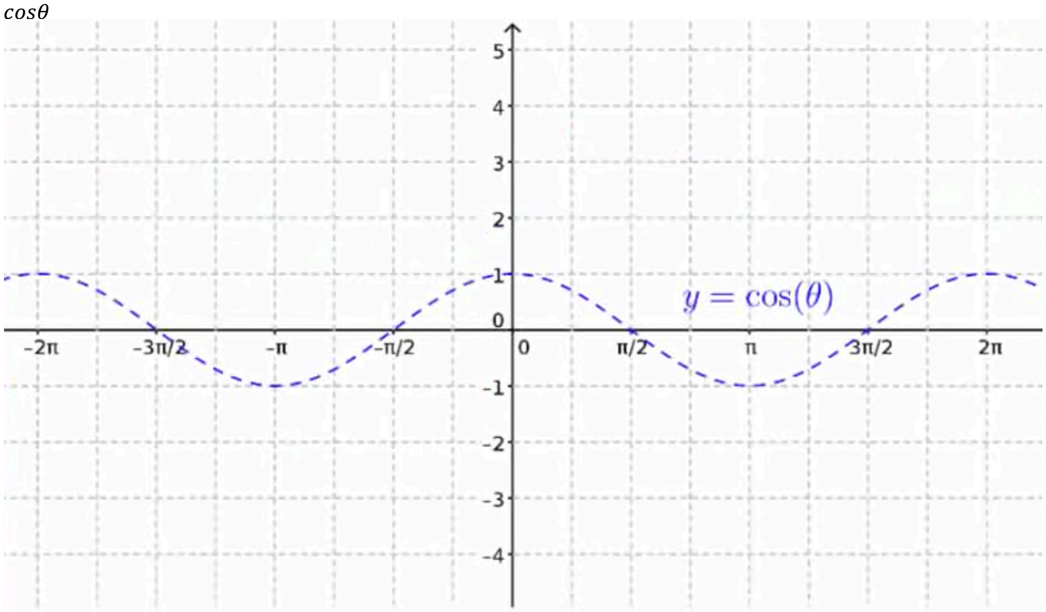
Lesson 6.5: Graphs of reciprocal Trig functions

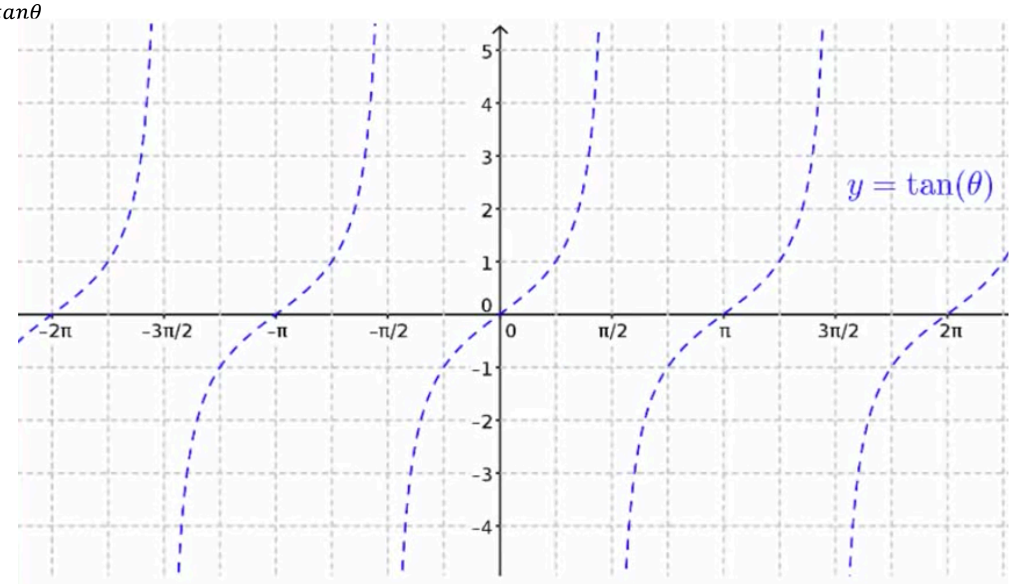
Graph: $csc\theta = \frac{1}{sin\theta}$



Graph: $sec\theta = \frac{1}{cos\theta}$



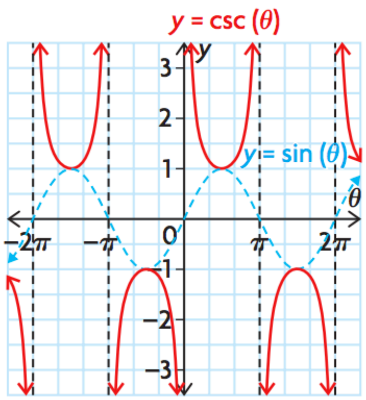
Graph: $cot\theta = \frac{1}{tan\theta}$



Summary:

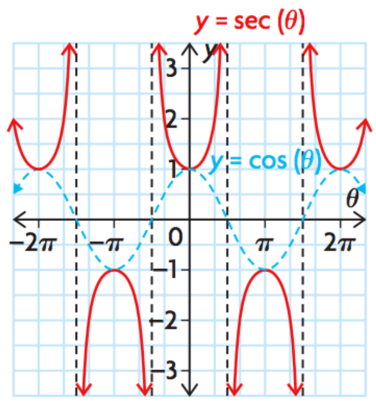
Characteristics	Cosecant	Secant	Cotangent
Domain			
Range			
Zeros			
Asymptotes			
Period			

Cosecant



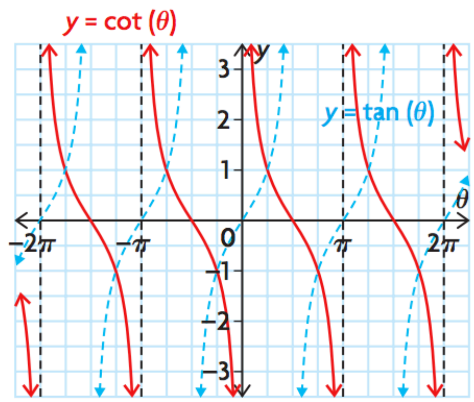
- has vertical asymptotes at the points where $\sin \theta = 0$
- has the same period (2π) as $y = \sin \theta$
- has the domain $\{x \in \mathbf{R} \mid \theta \neq n\pi, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R} \mid |y| \geq 1\}$

Secant



- has vertical asymptotes at the points where $\cos \theta = 0$
- has the same period (2π) as $y = \cos \theta$
- has the domain $\{x \in \mathbf{R} \mid \theta \neq (2n - 1)\frac{\pi}{2}, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R} \mid |y| \geq 1\}$

Cotangent



- has vertical asymptotes at the points where $\tan \theta = 0$
- has zeros at the points where $y = \tan \theta$ has asymptotes
- has the same period (π) as $y = \tan \theta$
- has the domain $\{x \in \mathbf{R} \mid \theta \neq n\pi, n \in \mathbf{I}\}$
- has the range $\{y \in \mathbf{R}\}$