Unit 4 – Trigonometric Identities and Solving Trig Equations
Chapter 7.1: Equivalent Trigonometric Functions

Due to the periodic nature of trigonometric functions, there are multiple ways to express equivalent functions.

1) Using the period:

Both sine and cosine have a period of 2π, which means any phase shift by a multiple of the period will be equivalent.

$sinx = sin(x + 2\pi) = sin\ (x - 2\pi)$

$cosx = cos(x + 2\pi) = cos\ (x - 2\pi)$

$cscx = csc(x + 2\pi) = csc\ (x - 2\pi)$

$secx = sec(x + 2\pi) = sec\ (x - 2\pi)$

For tangent and cotangent, $tanx = tan(x + \pi) = tan\ (x - \pi)$ and $cotx = cot(x + \pi) = cot\ (x - \pi)$

2) By symmetry:

Recall, even functions: $f(x) = f(-x)$

odd functions: $f(-x) = -f(x)$

Cosine and Secant are even (reflective symmetry across the y-axis):

$cosx = cos(-x)\,, sec(-x) = -secx$

Sine, cosecant, tangent, and cotangent are odd (rotational symmetry):

$sin(-x) = -sinx,\ csc(-x) = -cscx, tan(-x) = -tanx, cot(-x) = -cotx$

3) Using C.A.S.T rule:

Recall: you can identify equivalent trigonometric expressions by comparing principal angles drawn in standard position in quadrants II, III, and IV with their related acute angle, θ, in quadrant I.

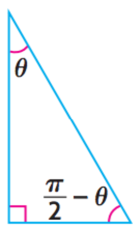
Principal Angle in Quadrant II	Principal Angle in Quadrant III	Principal Angle in Quadrant IV
$\sin(\pi - \theta) = \sin\theta$	$\sin(\pi + \theta) = -\sin\theta$	$\sin(2\pi - \theta) = -\sin\theta$
$\cos(\pi - \theta) = -\cos\theta$	$\cos(\pi + \theta) = -\cos\theta$	$\cos(2\pi - \theta) = \cos\theta$
$\tan(\pi - \theta) = -\tan\theta$	$\tan(\pi + \theta) = \tan\theta$	$\tan(2\pi - \theta) = -\tan\theta$

4) Using complimentary angles:

Recall: Complimentary angles add to $\frac{\pi}{2}$ (or 90°)

$sin\frac{\pi}{3} =$	$cos\frac{\pi}{6} =$	$csc\frac{\pi}{3} =$	$sec\frac{\pi}{6} =$
$cos\frac{\pi}{3} =$	$sin\frac{\pi}{6} =$	$sec\frac{\pi}{3} =$	$csc\frac{\pi}{6} =$
$tan\frac{\pi}{3} =$	$cot\frac{\pi}{6} =$	$cot\frac{\pi}{3} =$	$tan\frac{\pi}{6} =$

Hence: The above pattern is defined by Cofunction Identities by which describe trigonometric relationships between the complementary angles θ and $(\frac{\pi}{2} - \theta)$ in a right triangles.



$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$
$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$
$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

Example 1: Use R.A.A and cofunction identities to write an expression that is equivalent to each of the following expressions.

a) $\sec\frac{2\pi}{3} =$

b) $\tan\frac{7\pi}{6} =$

Practice:

Use the cofunction identities to write an expression that is equivalent to each of the following expressions.

a) $\sin \frac{\pi}{6}$ **c)** $\tan \frac{3\pi}{8}$ **e)** $\sin \frac{\pi}{8}$

b) $\cos \frac{5\pi}{12}$ **d)** $\cos \frac{5\pi}{16}$ **f)** $\tan \frac{\pi}{6}$

Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin \frac{7\pi}{8}$ **c)** $\tan \frac{5\pi}{4}$ **e)** $\sin \frac{13\pi}{8}$

b) $\cos \frac{13\pi}{12}$ **d)** $\cos \frac{11\pi}{6}$ **f)** $\tan \frac{5\pi}{3}$

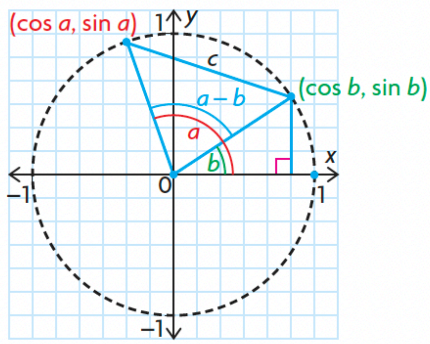
State whether each of the following are true or false. For those that are false, justify your decision.

a) $\cos (\theta + 2\pi) = \cos \theta$ **d)** $\tan (\pi - \theta) = \tan \theta$

b) $\sin (\pi - \theta) = -\sin \theta$ **e)** $\cot \left(\frac{\pi}{2} + \theta\right) = \tan \theta$

c) $\cos \theta = -\cos (\theta + 4\pi)$ **f)** $\sin (\theta + 2\pi) = \sin (-\theta)$

Chapter 7.2: Compound angle formulas



By the cosine law, $c^2 = 1^2 + 1^2 - 2(1)(1)\cos(a-b)$

① $c^2 = 2 - 2\cos(a-b)$

However, c has endpoints of $(\cos a, \sin a)$ and $(\cos b, \sin b)$.

By the distance formula, $c = \sqrt{(\sin a - \sin b)^2 + (\cos a - \cos b)^2}$

Squaring both sides,

$c^2 = (\sin a - \sin b)^2 + (\cos a - \cos b)^2$

$c^2 = \sin^2 a - 2\sin a \sin b + \sin^2 b + \cos^2 a - 2\cos a \cos b + \cos^2 b$

$c^2 = \sin^2 a + \cos^2 a - 2\sin a \sin b - 2\cos a \cos b + \sin^2 b + \cos^2 b$

$c^2 = 1 - 2\sin a \sin b - 2\cos a \cos b + 1$

② $c^2 = 2 - 2\sin a \sin b - 2\cos a \cos b$

Equating ① and ②,

$2 - 2\cos(a-b) = 2 - 2\sin a \sin b - 2\cos a \cos b$

Solving for $\cos(a-b)$,

$\cos(a-b) = \sin a \sin b + \cos a \cos b$

In Summary

Key Idea

- The trigonometric ratios of compound angles are related to the trigonometric ratios of their component angles by the following compound angle formulas.

Addition Formulas

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

Subtraction Formulas

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Example 1: Determine the exact value of

a) $cos\left(\frac{\pi}{12}\right)$

b) $tan\left(-\frac{5\pi}{12}\right)$

Example 2: Simplify each expression

a) $cos\frac{7\pi}{12}cos\frac{5\pi}{12}+sin\frac{7\pi}{12}sin\frac{5\pi}{12}$

b) $sin2xcosx-cos2xsinx$

Example 3: If a is an angle in quadrant I and b is an angle in quadrant II, evaluate $sin\,(a+b)$, where $sina=\frac{3}{5}$ and $sinb=\frac{5}{13}$.

Example 4:

Simplify $\frac{\sin(f+g) + \sin(f-g)}{\cos(f+g) + \cos(f-g)}$.