

Unit 4 – Trigonometric Identities and Equations

Chapter 7.3: Double angle formula

Key Idea

- The double angle formulas show how the trigonometric ratios for a double angle, 2θ , are related to the trigonometric ratios for the original angle, θ .

Double Angle Formula for Sine

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

Double Angle Formulas for Cosine

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

Double Angle Formula for Tangent

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Example 1: Simplify each of the following expressions and then evaluate

a) $2 \sin \frac{\pi}{8} \cos \frac{\pi}{8}$

b) $\frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$

Example 2: If $\cos \theta = -\frac{2}{3}$ and $0 \leq \theta \leq 2\pi$, determine the value of $\cos 2\theta$ and $\sin 2\theta$.

Practice: If $\tan \theta = -\frac{3}{4}$, where $\frac{3\pi}{2} \leq \theta \leq 2\pi$, calculate the value of $\cos 2\theta$.

Example 3: Develop a formula for $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$. (Half angle formula)

Practice:

1.

Jim needs to find the sine of $\frac{\pi}{8}$. If he knows that $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$, how can he use this fact to find the sine of $\frac{\pi}{8}$? What is his answer?

2.

Marion needs to find the cosine of $\frac{\pi}{12}$. If she knows that $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, how can she use this fact to find the cosine of $\frac{\pi}{12}$? What is her answer?