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Unit 6 – Exponential and Logarithmic functions

Lesson 3: Logarithmic law

Recall: Exponent laws

Same base	Different bases
$(a^x)(a^y) = a^{x+y}$	$(ab)^x = a^x b^x$
$(a^x) \div (a^y) = \frac{a^x}{a^y} = a^{x-y}, a \neq 0$	$(\frac{a}{b})^x = \frac{a^x}{a^y}. b \neq 0$
$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$	
$(a^x)^y = a^{xy}$	
$a^0 = 1, a \neq 0$	

Similarly, Log will also have a set of laws by which can make their calculation much easier.

$$\boldsymbol{log_a xy = log_a x + log_a y}$$

$$\boldsymbol{log_a\left(\frac{x}{y}\right) = log_a x - log_a y}$$

$$\boldsymbol{log_a x^r = r log_a x}$$

Example 1: simplify then evaluate.

a) $log_3 6 + log_3 4.5$

b) $log_2 48 - log_2 3$

c) $log_5 \sqrt[3]{25}$

Example 2: Rewrite as a single log to a common base: $log 12 + \frac{1}{2} log 7 - log 2$

Example 3: Compare and contrast the graphs of $f(x) = log\,(1000x)$ and $g(x) = 3 + log\,(x)$.

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Example 4: Use the properties of logarithms to express $log_a\sqrt{\frac{x^3y^2}{w}}$ in term of $log_a x, log_a y, log_a w$.

Practice from Textbook: **pg475**

2. Express each of the following as a logarithm of a product or quotient.

a) $\log 5 + \log 7$ **d)** $\log x - \log y$

b) $\log_3 4 - \log_3 2$ **e)** $\log_6 7 + \log_6 8 + \log_6 9$

c) $\log_m a + \log_m b$ **f)** $\log_4 10 + \log_4 12 - \log_4 20$

3. Express each of the following in the form $r \log_a x$.

a) $\log 5^2$ **c)** $\log_m p^q$ **e)** $\log_7 (36)^{0.5}$

b) $\log_3 7^{-1}$ **d)** $\log \sqrt[3]{45}$ **f)** $\log_5 \sqrt[5]{125}$

4. Use the laws of logarithms to simplify and then evaluate each expression.

a) $\log_3 135 - \log_3 5$ **c)** $\log 50 + \log 2$ **e)** $\log_2 224 - \log_2 7$

b) $\log_5 10 + \log_5 2.5$ **d)** $\log_4 4^7$ **f)** $\log \sqrt{10}$

5. Describe how the graphs of $y = \log_2(4x)$, $y = \log_2(8x)$, and $y = \log_2\left(\frac{x}{2}\right)$ are related to the graph of $y = \log_2 x$.

6. Evaluate the following logarithms.

a) $\log_{25} 5^3$ **d)** $\log_2 \sqrt{36} - \log_2 \sqrt{72}$

b) $\log_6 54 + \log_6 2 - \log_6 3$ **e)** $\log_3 54 + \log_3\left(\frac{3}{2}\right)$

c) $\log_6 6\sqrt{6}$ **f)** $\log_8 2 + 3\log_8 2 + \frac{1}{2}\log_8 16$

7. Use the laws of logarithms to express each of the following in terms of $\log_b x$, $\log_b y$, and $\log_b z$.

a) $\log_b xyz$ **c)** $\log_b x^2 y^3$

b) $\log_b\left(\frac{z}{xy}\right)$ **d)** $\log_b \sqrt{x^5 y z^3}$

8. Explain why $\log_5 3 + \log_5 \frac{1}{3} = 0$.

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9. Write each expression as a single logarithm.

a) $3\log_5 2 + \log_5 7$

b) $2\log_3 8 - 5\log_3 2$

c) $2\log_2 3 + \log_2 5$

d) $\log_3 12 + \log_3 2 - \log_3 6$

e) $\log_4 3 + \frac{1}{2}\log_4 8 - \log_4 2$

f) $2\log 8 + \log 9 - \log 36$

10. A Use the laws of logarithms to express each side of the equation as a single logarithm. Then compare both sides of the equation to solve.

a) $\log_2 x = 2\log_2 7 + \log_2 5$

b) $\log x = 2\log 4 + 3\log 3$

c) $\log_4 x + \log_4 12 = \log_4 48$

d) $\log_7 x = 2\log_7 25 - 3\log_7 5$

e) $\log_3 x = 2\log_3 10 - \log_3 25$

f) $\log_5 x - \log_5 8 = \log_5 6 + 3\log_5 2$

11. Write each expression as a single logarithm. Assume that all the variables represent positive numbers.

a) $\log_2 x + \log_2 y + \log_2 z$

b) $\log_5 u - \log_5 v + \log_5 w$

c) $\log_6 a - (\log_6 b + \log_6 c)$

d) $\log_2 x^2 - \log_2 xy + \log_2 y^2$

e) $1 + \log_3 x^2$

f) $3\log_4 x + 2\log_4 x - \log_4 y$

12. Write $\frac{1}{2}\log_a x + \frac{1}{2}\log_a y - \frac{3}{4}\log_a z$ as a single logarithm. Assume that all the variables represent positive numbers.

13. Describe the transformations that take the graph of $f(x) = \log_2 x$ to the graph of $g(x) = \log_2(8x^3)$.

14. T Use different expressions to create two logarithmic functions that have the same graph. Demonstrate algebraically why these functions have the same graph.

15. C Explain how the laws of logarithms can help you evaluate $\log_3\left(\frac{\sqrt[5]{27}}{2187}\right)$.

Extending

16. Explain why $\log_x x^{m-1} + 1 = m$.

17. If $\log_b x = 0.3$, find the value of $\log_b x\sqrt{x}$.