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Teacher: Ms. Ella

Unit 6 – Exponential and Logarithmic functions

Chapter 8.5 – 8.6 Solving Exponential and Logarithmic Equations

Example 1: Solve the following exponential equations.

a) $\frac{9^{x-3}}{3^{4x+1}} = 81^{x+2}$

b) $3 \cdot 3^{2x} - 28 \cdot 3^{x} + 9 = 0$

c) $3^{x+2} = 4$ Practice: $5^{3x} = 30$

d) b) $2(6^{3x}) = 6(4^{2x-3})$ Practice: $27(2^{2x}) = 2(3^{x})$

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Example 2: Solve the following log equations with unknown base.

a) $log_x 0.04 = -2$ b) $log_x 10 = 20$

Example 3: Solve the following log equations.

a) $log_2 x - log_2 3 = log_2 6$ b) $log(x+2) + log(x-1) = 1$

Practice:

18. Solve the following system of equations algebraically.

$y = \log_2(5x + 4)$

$y = 3 + \log_2(x - 1)$

19. Solve each equation.

a) $\log_5(\log_3 x) = 0$ **b)** $\log_2(\log_4 x) = 1$

20. If $\left(\frac{1}{2}\right)^{x+y} = 16$ and $\log_{x-y} 8 = -3$, calculate the values of x and y.

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Challenge:

2018: only do part a)

8. (a) Determine all values of x such that $\log_{2x}(48\sqrt[3]{3}) = \log_{3x}(162\sqrt[3]{2})$.

2017: only do part b)

7. (a) Linh is driving at 60 km/h on a long straight highway parallel to a train track. Every 10 minutes, she is passed by a train travelling in the same direction as she is. These trains depart from the station behind her every 3 minutes and all travel at the same constant speed. What is the constant speed of the trains, in km/h?

(b) Determine all pairs (a, b) of real numbers that satisfy the following system of equations:

$$\sqrt{a} + \sqrt{b} = 8$$
$$\log_{10} a + \log_{10} b = 2$$

Give your answer(s) as pairs of simplified exact numbers.

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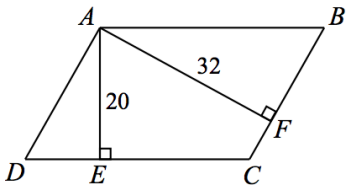
2016: only do part b)

8. (a) In the diagram, $ABCD$ is a parallelogram. Point E is on DC with AE perpendicular to DC, and point F is on CB with AF perpendicular to CB. If $AE = 20$, $AF = 32$, and $\cos(\angle EAF) = \frac{1}{3}$, determine the exact value of the area of quadrilateral $AECF$.



(b) Determine all real numbers $x > 0$ for which

$$\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$$

2015: only do part a)

9. Consider the following system of equations in which all logarithms have base 10:

$$\begin{aligned}(\log x)(\log y) - 3\log 5y - \log 8x &= a\\ (\log y)(\log z) - 4\log 5y - \log 16z &= b\\ (\log z)(\log x) - 4\log 8x - 3\log 625z &= c\end{aligned}$$

(a) If $a = -4$, $b = 4$, and $c = -18$, solve the system of equations.

(b) Determine all triples (a, b, c) of real numbers for which the system of equations has an infinite number of solutions (x, y, z).

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2014: only do part b)

7. (a) If $\dfrac{(x-2013)(y-2014)}{(x-2013)^2+(y-2014)^2} = -\dfrac{1}{2}$, what is the value of $x+y$?

(b) Determine all real numbers x for which

$$(\log_{10} x)^{\log_{10}(\log_{10} x)} = 10\,000$$

2013: only do part b)

8. (a) Determine all pairs (a, b) of positive integers for which $a^3 + 2ab = 2013$.

(b) Determine all real values of x for which $\log_2(2^{x-1} + 3^{x+1}) = 2x - \log_2(3^x)$.

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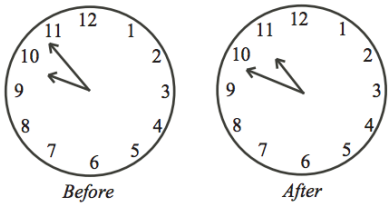
2012: only do part b)

8. (a) On Saturday, Jimmy started painting his toy helicopter between 9:00 a.m. and 10:00 a.m. When he finished between 10:00 a.m. and 11:00 a.m. on the same morning, the hour hand was exactly where the minute hand had been when he started, and the minute hand was exactly where the hour hand had been when he started. Jimmy spent t hours painting. Determine the value of t.



(b) Determine all real values of x such that

$$\log_{5x+9}(x^2+6x+9)+\log_{x+3}(5x^2+24x+27)=4$$

2011: only do part b)

7. (a) A 75 year old person has a 50% chance of living at least another 10 years.
A 75 year old person has a 20% chance of living at least another 15 years.
An 80 year old person has a 25% chance of living at least another 10 years.
What is the probability that an 80 year old person will live at least another 5 years?

(b) Determine all values of x for which $2^{\log_{10}(x^2)}=3(2^{1+\log_{10}x})+16$.

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2010

7. (a) Determine all real values of x for which $3^{(x-1)}9^{\frac{3}{2x^2}} = 27$.

(b) Determine all points (x, y) where the two curves $y = \log_{10}(x^4)$ and $y = (\log_{10} x)^3$ intersect.

2008

9. (a) The equation $2^{x+2}5^{6-x} = 10^{x^2}$ has two real solutions. Determine these two solutions.

(b) Determine all real solutions to the system of equations

$$\begin{aligned} x &+ \log_{10} x &&= y - 1 \\ y &+ \log_{10}(y-1) &&= z - 1 \\ z &+ \log_{10}(z-2) &&= x + 2 \end{aligned}$$

and prove that there are no more solutions.

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Extra Practice:

e) Bacteria are tripling. In 5 days, the population has quadrupled. What is the tripling time? Estimate the answer to 2 decimal places. Check your answer!

f) If $1 - 5^{2x+3} = 0$, find the value of x.

g) If $3^y = 5$, then determine the value of $3^{4y} - 11(3^{2y})$.

h) For the graph $f(x) = 2(2)^x$, you could say that it has vertically stretched $y = 2^x$. You could also get the same graph by shifting left 1 unit. Show how those two transformations result in the same graph, by altering the equation.

i) Write two different exponential functions that have an asymptote of $y = 5$.

j) For $f(x) = a(b)^x$,

k) What values should you have for a and b for it to be an increasing function?

l) Write two different equations of the form $f(x) = a(b)^x$ that are increasing functions.

m) Write two different equations of the form $g(x) = a(b)^x$ which are decreasing functions

n) For $y = b^{kx}$,

o) What values should you have for b and k to make it decreasing?

p) Write two different equations of the form $y = b^{kx}$ that are decreasing.

q) Write two different equations of the form $y = b^{kx}$ that are increasing.

r) Solve $4^{x^2+2} = 4^{3x}$

s) Solve $2^{x^2+5x} = \frac{1}{64}$

t) Solve $6^{x+1} + 6^{x+2} = 7$

u) Solve $3(3^{2x}) - 10(3^x) + 3 = 0$

v) Solve $2^{2x} - 2^{x+1} - 8 = 0$

w) Simplify $\sqrt{\sqrt[3]{\sqrt[4]{x}}}$

x) Simplify $\sqrt{x\sqrt{x\sqrt{x}}}$

y) Simplify $\sqrt[3]{x\sqrt{x\sqrt[4]{x}}}$

z) Solve the following system of equations:

$$2^{2x+y} = 32$$
$$2^{x-3y} = \frac{1}{2}$$

aa) If $2^{3x} = 16^{y+1}$ and $2x = 5y - 17$, determine the value of $x + y$. Don't use a calculator.

bb) Are there values of x that make the statement $x^{-2} > x^2$ true?

cc) Let $x = m + n$, where m and n are positive integers satisfying the equation $2^6 + m^n = 2^7$. Find the sum of all possible values of x.

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