MHF 4U – Exam Review

Part I – Polynomial Functions:

1. Determine the zeroes of the function $f(x)=3(x^2-25)(4x^2+4x+1)$.

2. Determine the equation of:
   a. The cubic function with zeroes 1, -2, -5 and y-intercept 10.
   b. The quartic function with roots at 2 & -1, and a double root at 3, and through (4,-10)

3. By calculating the Finite Differences, determine
   a. Which type of polynomial function best models this data.
   b. Determine the minimum and maximum number of zeroes and turning points for this type of function.

x	-2	-1	0	1	2	3	4
$f(x)$	1	0	1	16	81	256	625

4. a) Find the remainder when $2x^3-4x+7$ is divided by $2x-1$
   b) Find k if $(x-5)$ is a factor of $4x^3-5x+kx-2$.
   c) Divide $(3y^3-2y^2+12y-9)\div(y^2+2)$. State answer using a correct division statement.

5. Factor fully: a) $3x^3+8x^2+3x-2$ b) x^4-2x^3+2x-1

6. Solve for $x, x\in C$:
   a) $x^3-7x+6=0$ b) $2x^3+x^2-8x-4=0$ c) $9z^3-4z=0$
   d) $3x^3-7x^2=2-6x$ e) $x^2(2x-1)=2x-1$

7. Solve the following polynomial inequalities.
   a) $t^3-2t^2-3t>0$ b) $t^3+2t^2-3t<0$ c) $x(x-2)(x+1)(x+5)\geq 0$

8. For the function $g(x)=-2x^3(x-2)^2$
   a. State the degree of the function and comment on its end behaviour.
   b. State the zeroes and sketch the function.
   c. State from the graph, the interval(s) where $g(x)$ is decreasing.
   d. Find, algebraically where $g(x)<0$. Illustrate, in colour, on the graph.
   e. State the coordinates of any local/absolute max/min points.

Part II – Rational Functions

1. Find the asymptotes of $f(x)=\dfrac{x+2}{3x-2}$

2. Sketch the graph of $f(x)=\dfrac{x^2+4x+3}{x-2}$.

3. I) Given: $g(x)=\dfrac{x-2}{x^2+5x+6}$
   a. Determine x and y intercepts.
   b. State the domain.
   c. State any asymptotes.
   d. Are there any holes?

   ii) Given: h(x) = $\dfrac{x-3}{x^2-9}$
   a. Determine x and y intercepts.
   b. State the domain.
   c. Find all asymptotes.
   D. Are there any holes?

4. Create a function that has a graph with the given features.
   a. Vertical Asymptote at $x=-2$ and horizontal asymptote at $y=1$
   b. Two vertical asymptotes at $x=3$ and $x=-4$, and a hole at $x=1$
   c. Vertical Asymptote at $x=-3$, horizontal asymptote at $y=0$, no x-intercept and a y intercept at -2

5. Determine if the following functions have crossover points:

   a) $f(x)=\dfrac{(2-x)(3x+2)}{x^2}$ b) $f(x)=\dfrac{2x^3-3x^2+x-3}{x^2+1}$

6. Solve and state any restrictions.

   a. $\dfrac{x-1}{x-3}=\dfrac{x+3}{x+4}$ b. $\dfrac{x}{x-1}+\dfrac{1}{x+1}=\dfrac{2}{x^2-1}$ c. $0=2-\dfrac{10}{x^2}$

7. The estimate revenue and cost functions for the manufacture of a new product are R(x)=-2x^2+15x and C(x)=5x+8.
   a) Express the average profit function $AP(x)=\dfrac{P(x)}{x}$, in two different forms.
   b) Explain what can be determined from each form.
   c) What is the domain of the function in this context?
   d) What are the break-even quantities?

8. The value of a car, t years after it is bought, is modeled by $V(t)=\dfrac{2100+8t}{1+0.5t}+150$.
   a) What will the car be worth in the long run?
   b) Find the average rate of change in the value of the car between 2 and 5 years and the instantaneous rate of change at 2 years.

9. Sketch $f(x)=-x^2+9$ and its reciprocal on the same grid.

10. Solve a) $\dfrac{2(x-2)}{x+1}>0$ b) $\dfrac{3x}{x-2}>6$

Part III – Trigonometric Equations and Periodic Phenomena:

1. Sketch $-2\pi\le\theta\le2\pi$ of:

   c) y = 2 sin$3(\theta)$-1 b) y = -4 cos$\left(\dfrac{1}{2}\theta-\pi\right)+3$ c) y = $\dfrac{1}{2}$csc$\left(\theta+\dfrac{\pi}{4}\right)$

2. Given f(x) = sinx:
   a) Find the average rate of change (to 5 decimals) of the function from $\dfrac{\pi}{4}to\dfrac{3\pi}{4}$. What does this tell us about the graph?
   b) Find the instantaneous rate of change (to 5 decimals) of the function at $\dfrac{\pi}{6}$. What does this tell us about the graph?

3. State the RAA, principle angle and two co-terminal angles for each of the following.

   a) $\dfrac{2\pi}{3}$ b) $-\dfrac{3\pi}{4}$

4. Find each value.

a) $\csc\theta$ if $\sin\theta = \frac{3}{\sqrt{5}}$ b) $\cot\theta$ if $\csc\theta = \frac{7}{3}$

5. Find each of the following. Use exact values only.

a) $\sin\frac{7\pi}{4}$ b) $\sec\frac{2\pi}{3}$ c) $\cot\frac{11\pi}{6}$ d) $\cos\frac{3\pi}{4}$

6. Find all values for $0 \le \theta \le 2\pi$.

a) $\cos\theta = \frac{\sqrt{3}}{2}$ b) $\sec\theta = -\sqrt{2}$ c) $\cot\theta = -1$ d) $\tan\theta = \sqrt{3}$

7. Solve:

a) $\sin 2\theta = \frac{1}{2}$ b) $\cos\left(\frac{\theta}{2}\right) = -\frac{1}{2}$ c) $\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}$

8. Solve:

a) $\cos^2 x = \frac{3}{4}$ b) $2\sin^2 x + \sin x - 1 = 0$ c) $10\cos^2(2x) + 7\cos(2x) = 6$

d) $4\cos^2(2x) - 1 = 0$ e) $2\tan^2 x + \tan x - 3 = 0$ f) $6\cos^2 x - \sin x - 4 = 0$

g) $2\cos x = 1 - \sin^2 x$

9. Evaluate the following:

a) $\cos\left(\frac{5\pi}{6}\right)$ b) $\sec\left(\frac{11\pi}{3}\right)$ c) $\sin\frac{\pi}{12}$ d) $\tan\frac{11\pi}{12}$

10. Write as a single trig function.

a) $\sin A\cos B - \cos A\sin B$ b) $\frac{2\tan x}{1-\tan^2 x}$ c) $10\sin x\cos x$

d) $1 - 2\sin^2\left(\frac{2\theta}{3}\right)$ e) $-4\sin\frac{x}{2}\cos\frac{x}{2}$ f) $\frac{\tan 3x - \tan 4x}{1 - \tan 3x\tan 4x}$

11. Develop a formula for $\sin 3\theta$ in terms of $\sin\theta$.

12. If $\sin x = \frac{4}{5}$ and $\cos y = -\frac{12}{13}$, where $0 \le x \le \frac{\pi}{2}$ and $\frac{\pi}{2} \le y \le \pi$, find the following.

a) $\sin(x - y)$ b) $\tan(x + y)$

13. If $\cot x = -\frac{2}{3}$ for $\frac{\pi}{2} < x < \pi$, find the following.

a) $\tan 2x$ b) $\csc 2x$

14. Prove the following identities.

a) $\frac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$ b) $\frac{\cos^4 x - \sin^4 x}{1-\tan^4 x} = \cos^4 x$ c) $\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = 1 + \sin x\cos x$

d) $\dfrac{\cos(x-y)}{\cos x\cos y} = 1+\tan x\tan y$ e) $\dfrac{1-\cos 2x}{\tan x} = \sin 2x$ f) $2\cos x - 2\cos^3 x = \sin x \sin 2x$

g) $\cot\theta + \tan\theta = 2\csc 2\theta$

15. a) State the equations of the vertical asymptotes for
    i) y = sec x for $x \varepsilon \Re$
    ii) y = tan x for $x \varepsilon \Re$

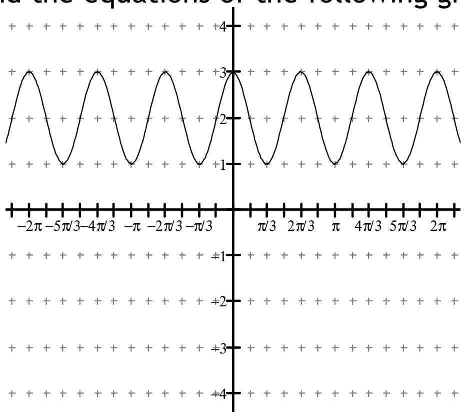
    b) State the period of y = tan 2x.

16. A ferris wheel, with a diameter of 40m, makes a full rotation in 3 minutes. Passengers board at the bottom of the ferris wheel, which is 2m above ground.

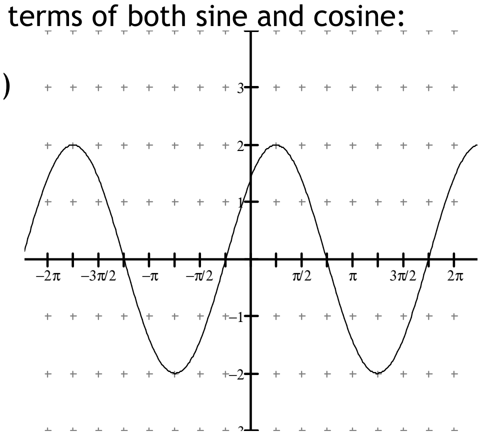
    a) Find the equation that best models the height of a passenger.
    b) Find the height of a passenger who has been riding the ferris wheel for 150 seconds.

17. Find the equations of the following graphs in terms of both sine and cosine:

a)



b)



Part IV - Exponential and Logarithmic Functions:

1. Sketch the following functions.
   (a) $y = 3(2)^x$ (b) $y = -(2)^{3x}$ (c) $y = 2^{-x} + 4$

2. Sketch the following functions.
   (a) $y = -\log x + 2$ (b) $y = \log\ (x+4)$ (c) $y = -\log(-2x) + 2$

3. Express in logarithmic form
   (a) $5^2 = 25$ (b) $6^0 = 1$ (c) $49^{\frac{1}{2}} = 7$ (d) $125^{\frac{2}{3}} = 25$

4. Express in exponential form
   (a) $\log_3 1 = 0$ (b) $\log_6\left(\frac{1}{36}\right) = -2$ (c) $\log_9 27 = \dfrac{3}{2}$

5. Evaluate the following
   (a) $\log_7 49$ (b) $\log_2 16$ (c) $\log_2 \sqrt[3]{32}$ (d) $6^{-2\log_6 8}$
   (f) $\log_4(64\times\sqrt[3]{16})$ (e) $\log_{12} 576 - \log_{12} 4$ (f) $\log_6 18 + \log_6 12$

6. Solve
   (a) $\log x = \log 5 + 2\log 3$ (b) $\log_3 x - \log_3 4 = \log_3 12$
   (c) $\log_6(x+3) + \log_6(x-2) = 1$ (d) $\log_5(7x+1) - \log_5(x-1) = 2$

7. Express as a single log:

(a) $\frac{1}{2}\log_5 x+\frac{1}{3}\log_5 y-\frac{1}{4}\log_5 z$ (b) $\frac{1}{2}[\log_4 x+3\log_4 y]-2[\log_4 a+\log_4 b]$

8. Solve

(a) $3^{2x-1}=5$ (b) $2^{2x}+3(2^x)-10=0$ (c) $\left(\frac{1}{64}\right)^{x+2}=8^{2x}$ d) $6(10^{2x})+10^x-2=0$

9. A sample of 500 cells in a medical research lab doubles every 20 min.
   a) Determine a formula for the number of cells at time t, where t is measured in minutes.
   b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places.

10. A sample of radioactive iodine-131 atoms has a half-life of about 8 days. Suppose that 1 000 000 iodine-131 atoms are initially present.
    a) Determine a formula for the number of atoms at time t, where t represents number of days
    b) How long will it take for the sample to reach 180 000 atoms? Answer correct to 2 decimal places.

11. A new car depreciates in value by 4.5% every year. When will the car be worth half its original value?

12. Most of Canada's earthquakes occur along the west coast. In 1949, there was an earthquake in the Queen Charlotte Islands that had a magnitude of 8.1 on the Richter Scale. In 1997 there was an earthquake in south-western B.C. with a magnitude of 4.6 on the Richter Scale. How many times as intense as the 1997 earthquake was the 1949 earthquake? Answer correct to 2 decimal places.

13. The loudness level of a heavy snore is 69 dB. How many times is this more intense than conversational speech at 60 dB? Answer correct to 2 decimal places.

14. A liquid has a pH of 4.5. What is the hydrogen ion concentration (mol/L) in the liquid?

15. Given the function $y=4^x$, determine:
    a. The average rate of change from t=5 seconds to t=6 seconds.
    b. The instantaneous rate of change at t=5 seconds.

Part V – Combination of Functions:

1. Given $f(x) = 2^x$ and $g(x) = \log_5 x$, find the domain of: a) (f + g)(x) b) (f x g)(x) c) (f/g)(x)

2. Given f(x) = x + 3 and $g(x) = x^2$, sketch the graphs of: a) (f - g)(x) b) (f x g)(x)

3. Given $f(x) = x^2 - 4$ and $g(x) = \frac{1}{x}$, find: a) f(g(2)) b) g(f(2)) c) f(g(x)) d) g(f(x))

4. A spherical hailstone grows in a cloud. The hailstone maintains a spherical shape while its radius increases at a rate of 0.2 mm/min

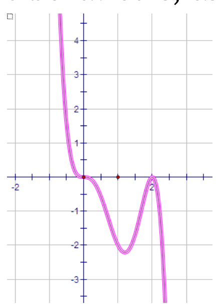
   a) Express the radius, r, in millimetres, of the hailstone, as a function of the time, t, in minutes.
   b) Express the volume, V, in cubic millimetres, of the hailstone, in terms of r.
   c) Determine $(V \circ r)(t)$ and explain what it means.
   d) What is the volume of the hailstone 1h after it begins to form?

Answers:

Part I - Polynomial Functions:

1. 5, -5, $\frac{-1}{2}$
2. a) f(x) = $-x^3 - 6x^2 - 3x + 10$ b) f(x) = $-x^4 + 7x^3 - 13x^2 - 3x + 18$
3. a) The 4th differences are constant (value is 24). Therefore it is quartic.
   b) Minimum number of zeroes is 0, Maximum number of zeroes is 4, minimum number of turning points is 1, maximum number of turning points is 3
4. a) 5.25   b) –94.6   c) $3y^3 - 2y^2 + 12y - 9 = (y^2 + 2)(3y - 2) + (6y - 5)$
5. a) $(x+1)(3x-1)(x+2)$ b) $(x-1)^3(x+1)$
6. a) $x = 1, 2, -3$ b) $x = \pm 2, \frac{-1}{2}$ c) $z = 0, \frac{\pm 2}{3}$ d) $x = 1$, $x = \frac{2 \pm \sqrt{2}i}{3}$ e) $x = \pm 1, \frac{1}{2}$
7. a) $-1 < t < 0$, $t > 3$ b) $0 < t < 1$, $t < -3$ c) $x \le -5$, $-1 \le x \le 0$, $x \ge 2$
8. a) Degree is 5 , For end behaviours, as $x \to +\infty, y \to -\infty$ and as $x \to -\infty, y \to +\infty$

   b) Zeroes are 0 and 2



   c) The function is decreasing when $x < 1.2$ (approximately from graph) and when $x > 2$

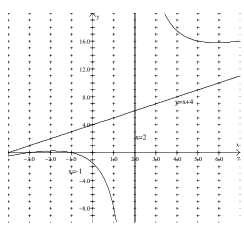
   d) $g(x) < 0$ when $0 < x < 2$ and $x > 2$

   e) Local minimum at approximately (1.2, -2.2) and local maximum at (2,0)

Part II - Rational Functions:

1. VA: $x = \frac{2}{3}$ HA: $y = \frac{1}{3}$

2.



3. I) a) x-intercept is 2 and y-intercept is $\frac{-1}{3}$ b) Domain D= $\{x | x \in R, x \ne -3, x \ne -2\}$
   c) VA: $x = -3, x = -2$ HA: $y = 0$ d) There are no holes.

   II)a) There is no x-intercept and y-intercept is $\frac{1}{3}$ b) Domain D = $\{x | x \in R, x \ne \pm 3\}$
   c) VA: $x = -3$ HA: $y = 0$ d) There is a hole at $x = 3$.

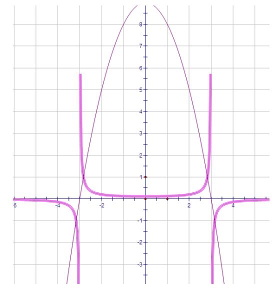
4. a) $y = \frac{x}{x+2}$ b) $y = \frac{x-1}{(x-1)(x-3)(x+4)}$ c) $y = \frac{-6}{(x+3)}$
5. a) (-1, -3)   b) (0, -3)

6. a) $x = \frac{-5}{3}$, $x \neq 3, -4$ b) $x = -3$ or $x = 1$, $x \neq \pm 1$ c) $x = \pm\sqrt{5}$, $x \neq 0$

7. a) $AP(x) = \frac{-2(x-4)(x-1)}{x}$ or $AP(x) = -2x + 10 - \frac{8}{x}$

b) From the factored form, you can find zeros and from the form $g(x) + \frac{1}{h(x)}$ you can find oblique asymptotes c) The Domain is: D = $\{x | x > 0, x \in R\}$ d) $x = 4, x = 1$

8. a) Worth $166 in long run
b) Average Rate of Change between t=2 and t=5 is: -$147.86.
Instantaneous Rate of Change at t=2 years is -$260.50

9. 

Part III – Trigonometric Equations and Periodic Phenomena

2. a) ARC = 0. b)IR C = 0.87

3. a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}, -\frac{4\pi}{3}$ b) $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, -\frac{11\pi}{4}$

4. a) $\frac{\sqrt{5}}{3}$ b) $\frac{2\sqrt{10}}{3}$

5. a) $-\frac{1}{\sqrt{2}}$ b) -2 c) $-\sqrt{3}$ d) $-\frac{1}{\sqrt{2}}$

6. a) $\frac{\pi}{6}, \frac{11\pi}{6}$ b) $\frac{3\pi}{4}, \frac{5\pi}{4}$ c) $\frac{3\pi}{4}, \frac{7\pi}{4}$ d) $\frac{\pi}{3}, \frac{4\pi}{3}$

7. a) $\frac{\pi}{12} + n\pi, \frac{5\pi}{12} + n\pi$ b) $\frac{4\pi}{3} + 4n\pi$ c) $\frac{4\pi}{3} + 2n\pi, n\pi$

8. a) $\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$ b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ c) $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

d) $\frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi, \frac{5\pi}{6} + n\pi$ e) $\frac{\pi}{4} + n\pi, 2.16 + n\pi$ f) $\frac{\pi}{6}, \frac{5\pi}{6}, 3.87, 5.55$ g) $\frac{\pi}{2} + n\pi$

9. a) $-\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$ b) $\csc\frac{\pi}{6} = 2$ c) $\frac{\sqrt{6} - \sqrt{2}}{4}$ d) $2 + \sqrt{3}$

10. a) sin (A - B) b) tan(2x) c) 5 sin(2x) d) $\cos\frac{4\theta}{3}$ e) -2sinx f) -tanx

11. $3\sin\theta - 4\sin^3\theta$ 12. (a) $-\frac{63}{65}$ (b) $\frac{33}{56}$ 13. (a) $\frac{12}{5}$ (b) $-\frac{13}{12}$

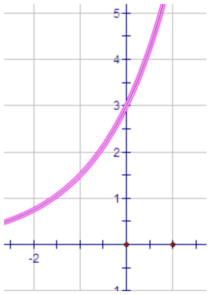
15. (a) (i) x = $(2n+1)\frac{\pi}{2}, n \in I$ (ii) x = $(2n+1)\frac{\pi}{2}, n \in I$ (b) Period is $\frac{\pi}{2}$

16. h(t) = -20 cos $\left(\frac{\pi}{3}t\right)$ +22, 39.32m

17. a) $y = \frac{1}{2}\sin\left(3\left(x - \frac{\pi}{2}\right)\right) + 2$ *or* $y = \frac{1}{2}\cos(3x) + 2$ b) $y = 2\sin\left(x - \frac{7\pi}{4}\right)$ *or* $y = 2\cos\left(x - \frac{\pi}{4}\right)$

Part IV – Exponential and Logarithmic Functions:

1. (a)



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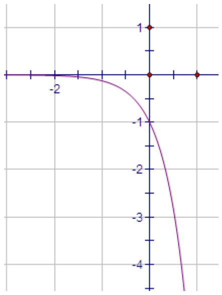

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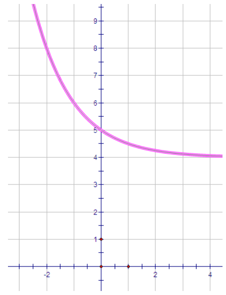
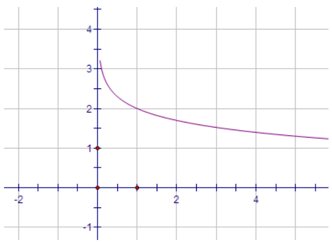


2. (a)



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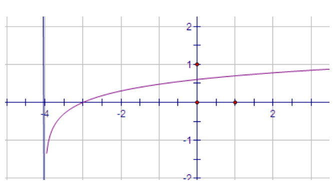

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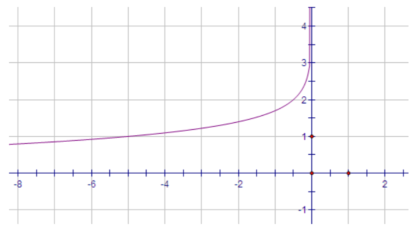


3. (a) $\log_5 25 = 2$ (b) $\log_6 1 = 0$ (c) $\log_{49} 7 = \frac{1}{2}$ (d) $\log_{125} 25 = \frac{2}{3}$

4. (a) $3^0 = 1$ (b) $6^{-2} = \frac{1}{36}$ (c) $9^{\frac{2}{3}} = 27$

5. (a) 2 (b) 4 (c) $\frac{3}{2}$ (d) $\frac{5}{3}$ (e) 0 (f) -4 (g) $\frac{1}{64}$ (h) 36 (i) $\frac{7}{2}$ (j) $\frac{11}{3}$ (k) $\frac{-10}{3}$
(l) 2 (m) 3

6. (a) $x = 45$ (b) $x = \frac{1}{64}$ (c) $x = 48$ (d) $x = 3$ (e) 64 (f) $\frac{13}{9}$

7. (a) $\log\left(\frac{\sqrt{x}\sqrt[3]{y}}{\sqrt[4]{z}}\right)$ (b) $\log_4\left(\frac{\sqrt{xy^3}}{a^2b^2}\right)$

8. (a) $x = 1.23$ (b) $x = 1$ (c) $x = -1$ (d) $x = -0.3$

9. (a) $N(t) = 500(2)^{\frac{t}{20}}$ (b) 103.40 min

10. (a) $N(t) = 1000000\left(\frac{1}{2}\right)^{\frac{t}{8}}$ (b) 19.79 days

11. 15.05 years

12. 3162.28

13. (a) 7.94

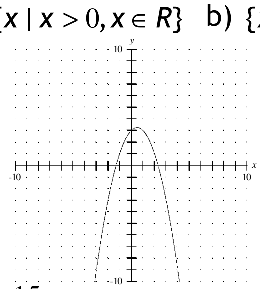
14. 3.16 x 10^{-5} mol/L

15. (a) 3072 (b) 1419.57

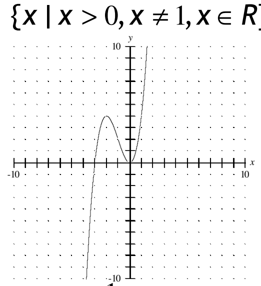
Part V - Combination of Functions:

1. a) $\{x \mid x > 0, x \in R\}$ b) $\{x \mid x > 0, x \in R\}$ c) $\{x \mid x > 0, x \neq 1, x \in R\}$

2. a)



b)



3. a) $-\frac{15}{16}$ b)DNE c) $f \circ g = \frac{1}{x^2} - 4$ d) $g \circ f = \frac{1}{x^2 - 4}$

4. a) r = 0.2t b) V = 4/3πr^3 c) V = 0.032/3πt^3 d) 7238 mm^3