# Polynomial Functions in Factored Form

For each of the following, state the zeros and the end behaviour.

a) $f(x) = (x + 7)(x + 4)(x - 5)$

- Zeros are -7, -4 and 5

- Function is cubic (degree 3)
- Leading coefficient is 1
- End behaviour:
  As $x \to \infty, \ f(x) \to \infty$
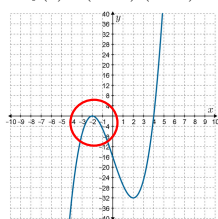  As $x \to -\infty, \ f(x) \to -\infty$

b) $f(x) = -2(x + 3)(x - 2)(2x - 5)^2$

- Zeros are -3, 2 and $\frac{5}{2}$

- Function is quartic (degree 4)
- Leading coefficient is -8
- End behaviour:
  As $x \to \infty, \ f(x) \to -\infty$
  As $x \to -\infty, \ f(x) \to -\infty$
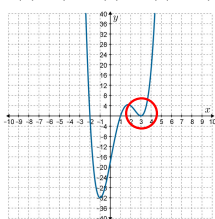
## Inspection

Inspect the graphs of $y = f(x)$ for the following functions. What do you notice about the zeros corresponding to squared factors?
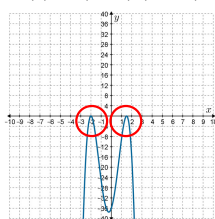
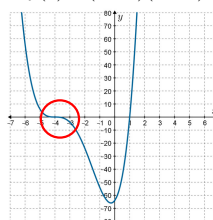$f(x) = (x + 2)^2(x - 4)$    $f(x) = (x + 2)(x - 1)(x - 3)^2$    $f(x) = -(x + 2)^2(2x - 3)^2$



The zeros corresponding to squared factors are also turning points. The graph has a parabolic shape near these points. The *x*-axis is tangent to the curve at these points.
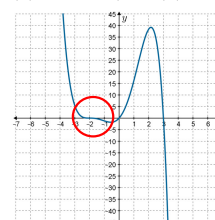
## Inspection

Inspect the graphs of $y = f(x)$ for the following functions. What do you notice about the zeros corresponding to cubed factors?
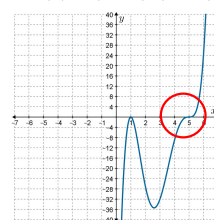
$f(x) = (x - 1)(x + 4)^3$    $f(x) = -0.3x(x - 3)(x + 2)^3$    $f(x) = (x - 1)^2(x - 5)^3$



The graph resembles the graph of $y = x^3$ at the zeros corresponding to cubed factors.

## Some examples…

The following are to be completed on a separate page.

1) Use zeros and end behaviour to sketch the graphs of the following functions.

   a) $f(x) = -0.5(x + 1)(x + 3)(x - 2)^2$      b) $f(x) = x^4 + 2x^3$

2) Write the equation of a cubic function that has zeros at -2, 3, and $\frac{2}{5}$ and that has a *y*-intercept of 6.

3) Write the equation of the quartic function shown in the graph on the right and state its domain and range.
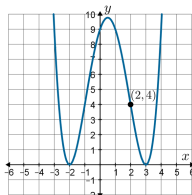


## A little bit of terminology…

**ORDER**:
The exponent to which a factor in an algebraic expression is raised is called its *order*. For example, in $f(x) = (x - 3)^2(x - 1)$, the order of $(x - 3)$ is 2 and the order of $(x - 1)$ is 1.

**FAMILY OF POLYNOMIAL FUNCTIONS**:

A *family of polynomial functions* is a set of polynomial functions whose equations have the same degree and whose graphs have common characteristics. For example, all functions of the form $y = a(x - 3)(x + 7)(x + 9)$, where $a \neq 0$.