Lesson #3: Properties of Limits

L.G.: "I can use the properties of limits to determine the limit of composite functions."

Side Note – Radical Expressions

Often times in math, we simplify radical expressions that appear in the denominator of an expression by **rationalizing** the denominator. To rationalize an expression is to eliminate the radical from the expression. To do this, we always multiply the expression to create a **difference of squares,** thus eliminating the radical in the expression! Let's try a few examples.

Example: Rationalize each of the following:

a) $2+\sqrt{2}$ b) $\sqrt{5}-\sqrt{7}$ c) $\frac{5}{\sqrt{3}}$ d) $\frac{-3}{2\sqrt{3}+3\sqrt{5}}$

Practice Questions: p9 2ac, 3ac, 4bc, 7

Properties of limits

For any real number **a**, such that f & g have limits that **exist** at x=a then

1. $\lim_{x\to a} k = k$, for any constant k
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
4. $\lim_{x\to a} [cf(x)] = c[\lim_{x\to a} f(x)]$, for any constant c
5. $\lim_{x\to a} [f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$
6. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided that $\lim_{x\to a} g(x) \neq 0$
7. $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$, for any rational number n

Example1: Using the **properties** of limits, evaluate the following:

a) $\lim_{x\to 2}(3x^2+4x-1)$ b) $\lim_{x\to -3}(2x^2-1)(3x+5)$ c) $\lim_{x\to 2}\frac{x^2-5x+2}{2x^3+3x+1}$ d) $\lim_{x\to -1}\sqrt{\frac{x^2}{x-5}}$

Required Practice: p45 #3, 4acf, **6**, **7-10ace, 10acd**

Example 2: Evaluate the following limits.

a) $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

b) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

c) $\lim_{x \to 0} \frac{(x+8)^{\frac{1}{3}} - 2}{x}$

d) $\lim_{x \to 2} \frac{|x-2|}{x-2}$

Required Practice: p45 #3, 4acf, **6**, **7-10ace, 10acd**