Lesson #4: Continuity of a Function

L.G.: "I can use limits to determine whether or not a function is continuous."

The concept of continuity can be thought of as the ability to draw a graph without lifting one's pencil. In other words, the graph must not contain any breaks or gaps.

Recall the Previous Examples: Discuss the continuity of the following piecewise functions by examining the limit:

a) $f(x)=\begin{cases}(x-1)^2+3, & x>1\\ x-1, & x\le 1\end{cases}$ Does the $\lim_{x\to 1} f(x)$ exist?

b) $f(x)=\begin{cases}\dfrac{x^2-2x-3}{x-3}, x\neq 3\\ -1, x=3\end{cases}$ Does the $\lim_{x\to 3} f(x)$ exist?

Definition of Continuity

A function is said to be **continuous** over an interval if, for ever value x = a in the interval,

1. The function has a value at x = a .
2. The limit of the function as $x\to a$ must exist. i.e. $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$
3. The two quantities in (1) and (2) are equal i.e. $f(a)=L$

Using the **three conditions** of **continuity,** determine whether or not the following functions are continuous:

a) $f(x)=\begin{cases}-3x-2, & x\ge 2\\ x^2-5, & x<2\end{cases}$

b) $f(x)=\begin{cases}\sqrt{x+2} & x>-2\\ x^2-4, & x\le -2\end{cases}$

c) $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3}, x\neq 3\\ 1, \ x=3\end{cases}$

Required Practice: p52 #4acf,**7**,8,**11**,13,**14,16**