#20: Verify that, if $f, g, h$ are differentiable, then

$$(fgh)'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

#21: a) Use the result in Exercise 20 to find the derivative of $F(x) = (x^2 + 1)[1 + (1/x)](2x^3 - x + 1)$.

b) Use the result in Exercise 20 to find the derivative of $G(x) = \sqrt{x}\,[1/(1 + 2x)]\,(x^2 + x - 1)$.

#22: Use Chain rule to differentiate the following.

a) $f(x) = (x - x^3 - x^5)^4$.

b) $f(t) = (t^2 - 1)^{100}$.

c) $f(t) = (t^{-1} + t^{-2})^4$.

d) $f(x) = \left(\dfrac{3x}{x^2 + 1}\right)^4$.

e) $f(x) = [(2x + 1)^2 + (x + 1)^2]^3$

f) $f(x) = \left(\dfrac{x^3}{3} + \dfrac{x^2}{2} + \dfrac{x}{1}\right)^{-1}$.

g) $f(x) = [(6x + x^5)^{-1} + x]^2$.

#23: Find $\dfrac{dy}{dx}$ at $x = 0$.

a) $y = \dfrac{1}{1 + u^2}$, $u = 2x + 1$.

b) $y = u + \dfrac{1}{u}$, $u = (3x + 1)^4$.

c) $y = \dfrac{2u}{1 - 4u}$, $u = (5x^2 + 1)^4$.

d) $y = u^3 - u + 1$, $u = \dfrac{1 - x}{1 + x}$.

#24: Find $\dfrac{dy}{dt}$.

a) $y = \dfrac{1 - 7u}{1 + u^2}$, $u = 1 + x^2$, $x = 2t - 5$

b) $y = 1 + u^2$, $u = \dfrac{1 - 7x}{1 + x^2}$, $x = 5t + 2$

#25: Given that

$f(0) = 1,\ f'(0) = 2,\ f(1) = 0,\ f'(1) = 1,$
$f(2) = 1,\ f'(2) = 1,$

$g(0) = 2,\ g'(0) = 1,\ g(1) = 1,\ g'(1) = 0,$
$g(2) = 2,\ g'(2) = 1,$

$h(0) = 1,\ h'(0) = 2,\ h(1) = 2,\ h'(1) = 1,$
$h(2) = 0,\ h'(2) = 2,$

evaluate the following.

1) $(f \circ g)'(0)$.
2) $(f \circ g)'(1)$.
3) $(f \circ g)'(2)$.
4) $(g \circ f)'(0)$.
5) $(g \circ f)'(1)$.
6) $(g \circ f)'(2)$.
7) $(f \circ h)'(0)$.
8) $(f \circ h \circ g)'(1)$.
9) $(g \circ f \circ h)'(2)$.
10) $(g \circ h \circ f)'(0)$.

#26: Find y''.

a) $f(x) = (x^2 - 5x + 2)^{10}$.
b) $f(x) = \left(\frac{x}{1-x}\right)^3$.
c) $f(x) = \sqrt{x^2+1}$

#27:. Find the indicated in terms of f'.

a) $\frac{d}{dx}[f(x^2+1)]$.
b) $\frac{d}{dx}\left[f\left(\frac{x-1}{x+1}\right)\right]$
c) $\frac{d}{dx}[[f(x)]^2 + 1]$.
d) $\frac{d}{dx}\left[\frac{f(x)-1}{f(x)+1}\right]$.

#28: An object is moving along the curve $y = x^3 - 3x + 5$ so that its $x$-coordinate at time $t$ is $x = 2t^2 - t + 2,\ t \geq 0$. At what rate is the $y$-coordinate changing when $t = 2$?

#29: An equilateral triangle of side length $x$ and altitude $h$ has area $A$ given by

$$A = \frac{\sqrt{3}}{4}x^2, \qquad \text{where} \quad x = \frac{2\sqrt{3}}{3}h.$$

Find the rate of change of $A$ with respect to $h$ and determine the rate of change of $A$ when $h = 2\sqrt{3}$.

#30: Air is being pumped into a spherical balloon in such a way that its radius is increasing at the constant rate of 2 centimeters per second. What is the rate of change of the balloon's volume at the instant the radius is 10 centimeters? (The volume $V$ of a sphere of radius $r$ is $V = \frac{4}{3}\pi r^3$.)

#71: If an object of mass $m$ has speed $v$, then its *kinetic energy*, KE, is given by

$$\mathrm{KE} = \tfrac{1}{2}mv^2.$$

Suppose that $v$ is a function of time. What is the rate of change of KE with respect to $t$?

#72: Newton's Law of Gravitational Attraction states that if two bodies are at a distance $r$ apart, then the force $F$ exerted by one body on the other is given by

$$F(r) = \frac{k}{r^2},$$

where $k$ is a positive constant. Suppose that, as a function of time, the distance between the two bodies is given by

$$r(t) = 49t - 4.9t^2, \qquad 0 \leq t \leq 10.$$

(a) Find the rate of change of $F$ with respect to $t$.
(b) Show that $(F \circ r)'(3) = -(F \circ r)'(7)$.

practice:

1. Determine $\frac{dy}{dx}$.

(a) $x^2 + y^2 = 36$

b) $x^3 - xy - y^2 = 13$

(c) $2x - y = x^3y^4$

2. Determine $\frac{dy}{dx}$.

(a) $\sqrt{x^3} + \sqrt[3]{y^2} = 7$

(b) $\sqrt{x} - \sqrt{y} = 5$

3. Determine an equation of the tangent to the curve at the given point. $x^3 + xy + y^2 = 3$, $(1, -2)$

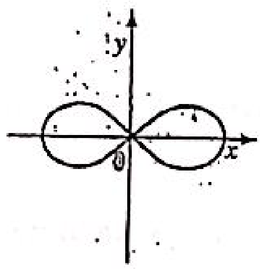
4. Find the point(s) on the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ where the slope of the tangent is equal to 1.

5. The curve with equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is called a *lemniscate* and is shown in the figure.

(a) Find $y'$.

(b) Find the equation of the tangent line to the lemniscate at the point $(-3, 1)$.

(c) Find the points on the lemniscate where the tangent line is horizontal.

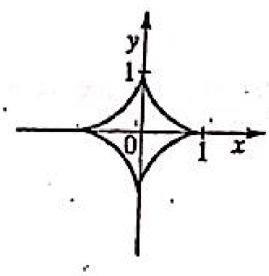



6. The curve with equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is called an *astroid* and is shown in the figure.

(a) Find $y'$.

(b) Find the equation of the tangent line to the astroid at the point $\left(\frac{1}{8}, \frac{3\sqrt{3}}{8}\right)$.

(c) Find the points on the astroid where the tangent line has slope 1.



Find the derivative of $y$ with respect to $x$ in each of the following.

(a) $y = \cos(-4x)$

(b) $y = \sin(3x + 2\pi)$

(c) $y = 4\sin(-2x^2 - 3)$

(d) $y = -\frac{1}{2}\cos(4 + 2x)$

(e) $y = \sin x^2$

(f) $y = -\cos x^2$

(g) $y = \sin^{-2}(x^3)$

(h) $y = \cos(x^2 - 2)^2$

(i) $y = 3\sin^4(2 - x)^{-1}$

(j) $y = x\cos x$

(k) $y = \dfrac{x}{\sin x}$

(l) $y = \dfrac{\sin x}{1 + \cos x}$

(m) $y = (1 + \cos^2 x)^6$

(n) $y = \sin\dfrac{1}{x}$

(o) $y = \sin(\cos x)$

(p) $y = \cos^3(\sin x)$

(q) $y = x\cos\dfrac{1}{x}$

(r) $y = \dfrac{\sin^2 x}{\cos x}$

(s) $y = \dfrac{1 + \sin x}{1 - \sin 2x}$

(t) $y = \sin^3 x + \cos^3 x$

(u) $y = \cos^2\left(\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)$

8. Find $\dfrac{dy}{dx}$ in each of the following.

(a) $\sin y = \cos 2x$

(b) $x\cos y = \sin(x + y)$

(c) $\sin y + y = \cos x + x$

(d) $\sin(\cos x) = \cos(\sin y)$

(e) $\sin x\cos y + \cos x\sin y = 1$

(f) $\sin x + \cos 2x = 2xy$

9. Find an equation of the tangent line to the given curve at the given point.

a) $y = \dfrac{\cos^2 x}{\sin^2 x}$ at $\left(\dfrac{\pi}{4}, 1\right)$ b) $y = \cos(\cos x)$ at $x = \dfrac{\pi}{2}$

10. Find the derivative of $y$ with respect to $x$ if
$x + \tan(xy) = \sin y + \cos x$

practice: Logarithmic method to take differentiation of complex function.

#1: Use logarithmic differentiation to find the derivative of each function.

(a) $y = (x^2 + 1)^2(x^2 + x + 1)^3$

(b) $y = (x - 1)^4(2x + 3)^5(x^2 - 2x + 3)^3$

(c) $y = e^{x^2}x^3(x^2 + 8)^4$

(d) $y = \dfrac{(x + 1)^3}{(x + 2)^5(x + 3)^7}$

(e) $y = \dfrac{x\sqrt{x + 1}}{(x + 2)(x^3 + 1)}$

(f) $y = \sqrt{\dfrac{x^2 + 1}{x^2 + 4}}$

#2: Differentiate.

(a) $y = x^{x^2}$

(b) $y = x^{\sqrt{x}}$

(c) $y = x^{\cos x}$

(d) $y = (\cos x)^x$

(e) $y = (\ln x)^x$

(f) $y = (\cos x)^{\sin x}$

#3: Find the equation of the tangent line to the curve $y = x^x$ at the point $(2, 4)$.

#4: In Problems 650 to 666 differentiate the given functions using the rules for logarithmic differentiation.

650. $y = x^{x^2}$.

651. $y = x^{x^x}$.

652. $y = (\sin x)^{\cos x}$.

653. $y = (\ln x)^x$.

654. $y = (x+1)^{2/x}$.

655. $y = x^3 e^{x^2} \sin 2x$.

656. $y = \dfrac{(x-2)^2 \sqrt[3]{x+1}}{(x-5)^3}$.

657. $y = x^{\ln x}$.

658. $y = \dfrac{(x+1)^3 \sqrt[4]{x-2}}{\sqrt[5]{(x-3)^2}}$.

659. $y = \sqrt{x \sin x \sqrt{1-e^x}}$.

661. $y = x^{\frac{1}{x}}$.

662. $y = x^{\sin x}$.

663. $y = \left(\dfrac{x}{1+x}\right)^x$.

664. $y = 2x^{\sqrt{x}}$.

665. $y = (x^2+1)^{\sin x}$.

666. $y = \sqrt[3]{\dfrac{x(x^2+1)}{(x^2-1)^2}}$.

#5: Find $\frac{dy}{dz}$:

a) $x + xy = e^y$  b) $2^x x - e^y x = y$

c) $y = \log_2(y+3x)$  ✓ d) $y = \ln(x^2+y^3)$

e) $xy = 5^{x^2+y^2}$  f) $\ln(x+y) = x^2+y^2$

#6. Use logarithmic differentiation to find $\frac{dy}{dx}$,

a) $y = \frac{(x+3)^3}{\sqrt{x^2-1}\,(x^2+3)^2}$  b) $y = \sqrt[3]{\frac{x^3-1}{3x^2+5x}}$

c) $y = \frac{x\sqrt[3]{x+1}}{(2x-3)^2(x^4+6)}$  d) $y = x^x$

e) $y = x^{\sqrt{x}+x}$  f) $y = (\ln x)^{x^2}$

g) $y = x^{\ln x}$  h) $y = x^{x^x}$

7: Use implicit differentiation to find $D_x y$.

1. $ye^{2x} + xe^{2y} = 1$  2. $e^x + e^y = e^{x+y}$  3. $e^y = \ln(x+y)$  4. $y^2e^{2x} + xy^3 = 1$

Unit 2 practice test: Derivative.

Fill in blank:

1. The derivative of $f(x) = e^{3x}$ is: ______

2. The derivative of $f(x) = 2^{\sin x}$ is: ______

3. The derivative of $f(x) = \cos x^2$ is: ______

4. The function $f'(x) = \frac{1}{2x}$ is the derivative function of: ______

5. The derivative of $f(x) = \sec x = \frac{1}{\cos x}$ is: ______

6. The derivative of $f(x) = \log(x^3)$ is: ______

7. The derivative of $f(x) = \sqrt{\sin x}$ is: ______

8. The derivative of $\log_2 f(x)$ is: ______

9. The derivative of $f(x) = x^2 \ln x$ is: ______

10. The second derivative of $f(x) = x \sin x$ is: ______

11. The equation of the tangent line to the curve $f(x) = 2^x$ at the point $P(0,1)$ is: ______

12. The derivative of $f(x) = \tan(x^2)$ is: ______

13. The tangent line to the curve $y = x^2 \ln x$ is horizontal at $x$ equal to: ______

14. if $f'(x) = 4f(x)$, then $f(x)$ is: ______

15. if the position function is $f(x) = 3\cos(2t)$, then the acceleration function is: ______

short answers:

1. Differentiate, simplify as much as you can.

a) $y = (1 + \log \sin x)^3$

b) $y = \sqrt[3]{\log_2 \tan \frac{x+3}{4}}$

c) $y = \dfrac{x^{\frac{5}{3}} \sqrt{x^4+2}}{(4x+1)^5}$

d) $y = (\sin x)^{\ln x}$

2. Use implicit differentiation to find and equation of the tangent to the curve $\sin(x+y) = 2x - 2y$ at the point $(\pi, \pi)$.

3. For the curve $x^2 + y^2 - xy + 3x - 9 = 0$,

a) where do the horizontal tangent line occur?

b) where do the vertical tangent lines occur?

4. Consider the equation $(\cos x)y^2 + (3(\sin x) - 1)y + 7x - 2 = 0$,

a) find $\frac{dy}{dx}$ at the point $(0, 2)$.

b) use the quadratic formula to solve for $y$ in terms of $x$.