pre-test

Fill in Blank:

1. The derivative of $f(x) = e^{3x}$ is:

$\boxed{3e^{3x}}$

2. The derivative of $f(x) = 2^{sinx}$ is:

$\boxed{2^{\sin x}\ln 2 \cos x}$

3. The derivative of $f(x) = \cos(x^2)$ is:

$-\sin x^2 \cdot 2x = \boxed{-2x\sin x^2}$

4. The function $f'(x) = \frac{1}{2x}$ is the derivative function of: $\frac{1}{2}\ln x$.

$\cancel{2\ln x}$ or $\boxed{\ln x^2}$

5. The derivative of $f(x) = \log(x^3)$ is:

$\frac{3x^2}{x^3 \cdot \ln 10} = \boxed{\frac{3}{x\ln 10}}$

6. The derivative of $f(x) = secx = \frac{1}{cosx}$ is:

$(\cos x)^{-1}$

$-(\cos x)^{-2} \cdot (-\sin x)$

$= \boxed{\frac{\sin x}{\cos^2 x}}$

7. The derivative of $f(x) = \sqrt{sinx}$ is:

$\frac{1}{2}(\sin x)^{-\frac{1}{2}} \cdot \cos x = \boxed{\frac{\cos x}{2\sqrt{\sin x}}}$

8. The derivative of $log_2 f(x)$ is:

$\boxed{\frac{f'(x)}{f(x)\ln 2}}$

9. The derivative of $f(x) = x^2 lnx$ is:

$y' = 2x\ln x + \frac{x^2}{x}$

$2x\ln x + x$

10. The second derivative of $f(x) = xsinx$ is:

$2\cos x - x\sin x$

~~$\cos x + x\sin x \cos x$~~

$y' = 1 \cdot \sin x + x\cos x$

$y'' = \cos x + \cos x + x(-\sin x)$

11. The equation of the tangent line to the curve $f(x) = 2^x$ at the point P(0, 1) is:

$y' = 2^x \cdot \ln 2$

$y'|_{x=0} = 2^0 \cdot \ln 2 = \ln 2$

$\therefore y = \ln 2 x + b$

$\because 1 = \ln 2 \cdot 0 + b$

$\therefore b = 1$

$\therefore y = \ln 2 x + 1$

$y = \ln 2 x + 1$

12. The derivative of $f(x) = \tan(x^2)$ is:

$y' = \sec^2 x^2 \cdot 2x$

$2x \sec^2 x^2$

13. The tangent line to the curve $y = x^2 lnx$ is horizontal at x equal to:

$y' = 2x = 0$

$x = 0$

$x = 0$ $e^{-0.5}$

14. If $f'(x) = 4f(x)$, then f(x) is:

$\because (e^{4x})' = e^{4x} \cdot 4 = f'(x)$

$\therefore f'(x) = 4f(x)$

e^{4x}

15. If the position function is $f(x) = 3\cos(2t)$ then the acceleration function is:

position $f(x)$

$\downarrow$

velocity $f'(x)$

$\downarrow$

acceleration $f''(x)$

$\therefore y' = -3\sin(2t) \cdot 2$

$= -6\sin 2t$

$y'' = -6\cos 2t \cdot 2$

$= -12\cos 2t$

$-12\cos 2t$

Short Answers:

16. Differentiate. Simplify as much as you can.

a) $y = (1 + logsinx)^n$

$y' = n(1 + \log \sin x)^{n-1} \cdot \left(0 + \frac{\cos x}{\sin x \cdot \ln 10}\right)$

$y' = n(1 + \log \sin x)^{n-1} \cdot \left(\frac{\cot x}{\ln 10}\right)$

b) $y = \sqrt[3]{log_2 sin\frac{x+3}{4}}$

$y' = \frac{1}{3}\left(\log_2 \sin\frac{x+3}{4}\right)^{-\frac{2}{3}} \cdot \frac{\cos\frac{x+3}{4} \cdot \frac{1}{4}}{\ln 2 \cdot \sin\frac{x+3}{4}}$

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17. Use logarithmic differentiation method to find the derivative of the following:

a) $y = \frac{x^{\frac{5}{3}}\sqrt{x^4+2}}{(4x+1)^5}$

$\ln y = \ln x^{\frac{5}{3}} + \ln \sqrt{x^4+2} - \ln (4x+1)^5$

$\ln y = \frac{5}{3}\ln x + \frac{1}{2}\ln x^4+2 - 5\ln(4x+1)$

$\frac{y'}{y} = \frac{5}{3}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{4x^3}{x^4+2} - \frac{5\cdot 4}{4x+1}$

$y' = \left(\frac{5}{3x} + \frac{4x^3}{2(x^4+2)} - \frac{20}{4x+1}\right)\cdot\frac{x^{\frac{5}{3}}\sqrt{x^4+2}}{(4x+1)^5}$

b) $y = \boxed{\sin(x)}^{\ln(x)}$

$\ln y = \ln\left(\sin x^{\ln x}\right)$

$\ln y = \ln x \cdot \ln \sin x$

$\frac{y'}{y} = \frac{1}{x}\ln \sin x + \ln x \cdot \frac{\cos x}{\sin x}$

$y' = \left(\frac{\ln \sin x}{x} + \frac{\ln x \cos x}{\sin x}\right)(\sin x)^{\ln x}$

18. A particle is moving according to the position function $s(t) = 2\sin t - \cos 2t$.
(Relations you may find useful: $\sin 2x = 2\sin x\cos x$ $\quad \cos 2x = \cos^2 x - \sin^2 x$)

a) Find the initial position of the particle.

$s(t=0) = 2\sin 0 - \cos 0 = 0 - 1 = -1$

b) Find the velocity function

$V(t) = s'(t) = 2\cos t + \sin 2t \cdot 2 = 2\cos t + 2\sin 2t$

c) Find the initial velocity of the particle.

$V(t=0) = 2\cos 0 + 2\sin 0 = 2 + 0 = 2$

d) Find the acceleration function.

$a(t) = v'(t) = -2\sin t + 2\cos 2t \cdot 2 = -2\sin t + 4\cos 2t$

e) Find the initial acceleration of the particle.

$a(t=0) = -2\sin 0 + 4\cos 0 = 0 + 4 = +4$ (~~slowing down~~)

f) Jerk is defined a derivative of acceleration with respect to time. Find the jerk function.

$J(t) = a'(t) = -2\cos t + 4(-\sin 2t)\cdot 2 = -2\cos t - 8\sin 2t$

g) Find the initial jerk of the particle.

$J(t=0) = -2\cos 0 - 8\sin 0 = -2 - 0 = -2$

19. Use implicit differentiation to find and equation of the tangent line to the curve $\sin(x+y) = 2x - 2y$ at the point (p, p).

(π, π)

Take derivative first:

$\therefore \cos(x+y) \cdot (1+y') = 2 - 2y'$

$\cos(x+y) + y'\cos(x+y) = 2 - 2y'$

$y'\cos(x+y) + 2y' = 2 - \cos(x+y)$

$y' = \dfrac{2 - \cos(x+y)}{\cos(x+y) + 2}$

plug in (π, π) to find "m".

$m = y'\Big|_{(\pi,\pi)} = \dfrac{2 - \cos 2\pi}{\cos 2\pi + 2} = \dfrac{2-1}{1+2} =$

$\therefore y = \frac{1}{3}x + b$

plug in (π, π) to find "b".

$\pi = \frac{1}{3}\pi + b$

$b = \frac{2}{3}\pi$

$\therefore \boxed{y = \frac{1}{3}x + \frac{2}{3}\pi}$

20. For the curve $x^2 + y^2 - xy + 3x - 9 = 0$:

a) Determine $\frac{dy}{dx}$.

$2x + 2yy' - (1 \cdot y + xy') + 3 - 0 = 0$

$2x + 2yy' - y - xy' + 3 = 0$

$y'(2y - x) = -2x + y - 3$

$y' = \dfrac{-2x + y - 3}{2y - x}$

b) Where do the horizontal tangent line occur?

when $y' = 0$

$\therefore -2x + y - 3 = 0$

$y = 2x + 3$

$(0, 3)$ & $(-4, -5)$

$\therefore$ For those y values where equal to $2x+3$ will have horizontal slope.

c) Where do the vertical tangent lines occur?

undefined

$\therefore$ when $2y - x = 0$

$y = \frac{1}{2}x$ put back to original

$(2, 1)$ & $(-6, -3)$

$\therefore$ For those x & y values that make $y = \frac{1}{2}x$ will have vertical tangent.

d) Determine $\frac{d^2y}{dx^2}$.

21. Consider the equation $(cosx)y^2 + (3sinx - 1)y + (7x - 2) = 0,$

a) Check that $x = 0, y = 2$ satisfies this equation.

$(\cos 0) \cdot 2^2 + (3\sin 0 - 1)(2) + (7 \cdot 0 - 2) = 0.$

$0 = 0.$

b) Find $\frac{dy}{dx}$ at the point $(0, 2)$.

c) Use the quadratic formula to solve for y in terms of x.

$(\cos x)y^2 + 3y\sin x - y + 7x - 2 = 0.$

$(-\sin x)y^2 + \cos x \cdot 2y \cdot y' + 3y'\sin x + 3y\cos x - y' + 7 - 0 = 0.$

plug in $(0,2)$:

$\cancel{(-\sin 0)(2)^2} + \cos 0 \cdot 2(2) \cdot y' + \cancel{3y'\sin 0} + 3(2)\cos 0 - y' + 7 = 0.$

$4y' + 6 - y' + 7 = 0$

$3y' = -13$

$\boxed{y' = -\frac{13}{3}}$

$a = \cos x$, $b = 3\sin x - 1$ $c = 7x - 2$

$\therefore \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ roots $= \frac{-(3\sin x + 1) \pm \sqrt{(3\sin x - 1)^2 - 4(\cos x)(7x-2)}}{2(\cos x)}$