7.2 Velocity

A Velocity
Velocity is a vector and the measurement unit is m/s or km/h.

Ex 1. Convert $5m/s$ into km/h.

$$5\frac{m}{s}=5\frac{\frac{1km}{1000}}{\frac{1h}{3600}}=5\frac{1km}{1000}\frac{3600}{1h}=5(3.6)km/h=18km/h$$

B Relative Velocity
The relative velocity of the object B traveling at $\vec{v}_B$ relative to the object A traveling at $\vec{v}_A$ is given by:

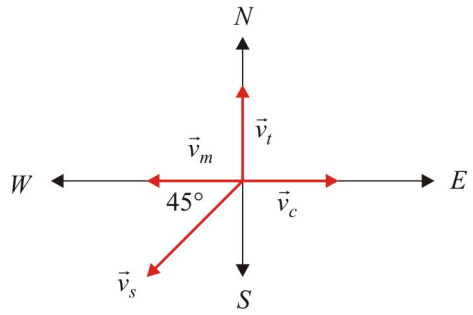
$$\vec{v}_{BA}=\vec{v}_B-\vec{v}_A$$

Note. If A is at rest ($\vec{v}_A=\vec{0}$) then:

$$\vec{v}_{BA}=\vec{v}_B$$

Ex 2. A car is traveling at $\vec{v}_c=100km/h[E]$, a motorcycle is traveling at $\vec{v}_m=80km/h[W]$, a truck is traveling at $\vec{v}_t=120km/h[N]$ and an SUV is traveling at $\vec{v}_s=100km/h[SW]$. Find the relative velocity of the car relative to:

a) motorcycle
b) truck
c) SUV



Let first change vectors into algebraic vectors:

$$\vec{v}_c=100km/h[E]=(100,0)km/h$$
$$\vec{v}_m=80km/h[W]=(-80,0)km/h$$
$$\vec{v}_t=120km/h[N]=(0,120)km/h$$
$$\vec{v}_s=100km/h[SW]=(-100\cos 45°,-100\sin 45°)$$
$$=(-50\sqrt{2},-50\sqrt{2})km/h$$

a) $\vec{v}_{cm}=\vec{v}_c-\vec{v}_m=(100,0)-(-80,0)=(180,0)km/h$

b) $\vec{v}_{ct}=\vec{v}_c-\vec{v}_t=(100,0)-(0,120)=(100,-120)km/h$

c) $\vec{v}_{cs}=\vec{v}_c-\vec{v}_s=(100,0)-(-50\sqrt{2},-50\sqrt{2})$
$=(100+50\sqrt{2},50\sqrt{2})km/h$

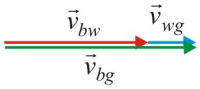
C Boat Velocity
The boat velocity relative to ground is vector sum between the boat velocity relative to water $\overrightarrow{v_{bw}}$ and the water velocity relative to ground $\overrightarrow{v_{wg}}$:

$$\overrightarrow{v_{bg}}=\overrightarrow{v_{bw}}+\overrightarrow{v_{wg}}$$

D Plane Velocity
The plane velocity relative to ground is vector sum between the plane velocity relative to air $\overrightarrow{v_{pa}}$ and the air velocity relative to ground $\overrightarrow{v_{ag}}$:

$$\overrightarrow{v_{pa}}=\overrightarrow{v_{pa}}+\overrightarrow{v_{ag}}$$

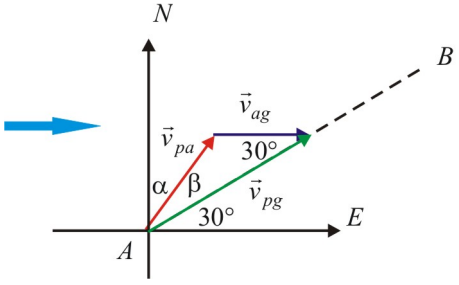
Ex 3. A river flows eastward with $4m/s$. A motorboat heads downstream the river between two towns which are $50km$ apart along the south bank of the river. If the motorboat speed in still water is $12m/s$, find:
a) the speed of the motorboat relative to the ground when traveling downstream



$$\vec{v}_{wg}=4m/s[E]$$
$$\vec{v}_{bw}=12m/s[E]$$
$$\vec{v}_{bg}=\vec{v}_{bw}+\vec{v}_{wg}=12m/s[E]+4m/s[E]=16m/s[E]$$
$$\therefore \|\vec{v}_{bg}\|=16m/s$$

Ex 4. A plane is scheduled to travel from the airport A to an airport B where $\overrightarrow{AB}=600km[060°]$. The speed of the plane relative to air is $300km/h$ and a strong wind of $100km/h$ is blowing eastward.

a) Draw a diagram to illustrate the situation.

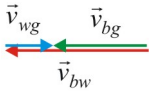


b) the time required to cover the distance between the towns downstream

$D = 50km$

$$t = \frac{D}{\| \vec{v}_{bg} \|} = \frac{50km}{16m/s} = 3125s = 52.08\min$$

c) the speed of the motorboat relative to the ground when traveling upstream



$\vec{v}_{wg} = 4m/s[E]$

$\vec{v}_{bw} = -12m/s[E]$

$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = -12m/s[E] + 4m/s[E] = -8m/s[E]$

$\therefore \| \vec{v}_{bg} \| = 8m/s$

d) the time required to cover the distance between the towns upstream

$$t = \frac{D}{\| \vec{v}_{bg} \|} = \frac{50km}{8m/s} = 6250s = 104.17\min$$

b) In what direction should the pilot head the plane?

$$\frac{\sin\beta}{\| \vec{v}_{ag} \|} = \frac{\sin 30°}{\| \vec{v}_{pa} \|} \Rightarrow \frac{\sin\beta}{100} = \frac{\sin 30°}{300} \Rightarrow$$

$$\beta = \sin^{-1}\left(\frac{\sin 30°}{3}\right) = 9.59°$$

$\alpha = 60° - 9.59° = 50.41°$

The pilot should head the plane in direction $[N50.41°E]$

c) What is the speed of the plane relative to ground?

$$\frac{\| \vec{v}_{pg} \|}{\sin(180° - 30° - 9.59°)} = \frac{\| \vec{v}_{pa} \|}{\sin 30°} \Rightarrow \frac{\| \vec{v}_{pg} \|}{\sin 140.41°} = \frac{300}{\sin 30°}$$

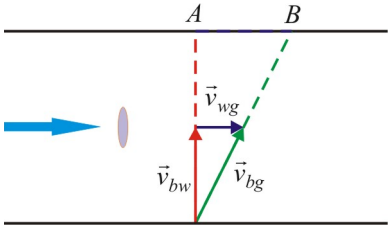
$$\| \vec{v}_{pg} \| = \frac{300\sin 140.41°}{\sin 30°} \cong 382.37km/h$$

d) How long will the trip last?

$$t = \frac{D}{\| \vec{v}_{pg} \|} = \frac{600km}{382.37km/h} \cong 1.57h$$

Ex 5. A river is $800m$ wide and flows eastward at $10m/s$. Peter is driving a motorboat heading always perpendicular to the current. The speed of the motorboat in still water is $20m/s$.

a) Draw a diagram to illustrate the situation.



b) What is the speed of the boat relative to ground?

$$\| \vec{v}_{bg} \| = \sqrt{\| \vec{v}_{bw} \|^2 + \| \vec{v}_{wg} \|^2} = \sqrt{20^2 + 10^2} = \sqrt{500}$$

$$\therefore \| \vec{v}_{bg} \| = 10\sqrt{5}m/s \cong 22.36m/s$$

c) How long does it take to cross the river?

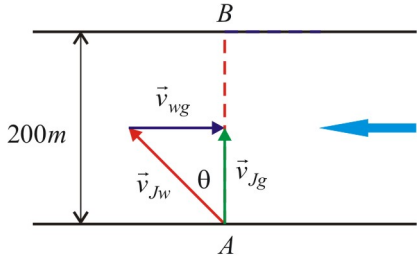
$$t = \frac{D}{\| \vec{v}_{bw} \|} = \frac{800m}{20m/s} = 40s$$

d) How much downstream does Peter reach the opposite bank?

$AB = \| \vec{v}_{wg} \| t = 10(40)m = 400m$

Ex 6. Jane can swim at $5m/s$ in still water. She wishes to swim across a river $200m$ wide to a point directly opposite from where she is standing. The river flows westward at $4m/s$ and she is standing on the South bank of the river.

a) Draw a diagram to illustrate the situation.



b) What is the speed of Jane relative to ground?

$$\| \vec{v}_{Jg} \| = \sqrt{\| \vec{v}_{Jw} \|^2 - \| \vec{v}_{wg} \|^2} = \sqrt{5^2 - 4^2} = 3m/s$$

c) In what direction must Jane head?

$$\sin\theta = \frac{\| \vec{v}_{wg} \|}{\| \vec{v}_{Jw} \|} = \frac{4}{5} \Rightarrow \theta = \sin^{-1}(4/5) = 53.13°$$

Jane must head into direction $[N53.13°W]$.

d) How long does it take to cross the river?

$$t = \frac{D}{\| \vec{v}_{Jg} \|} = \frac{200m}{3m/s} = 66.67s$$

Reading: Nelson Textbook, Pages 365-368
Homework: Nelson Textbook: Page 369 #1, 3, 4, 6, 9, 11, 13, 14