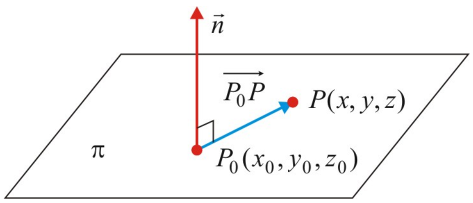
8.5 Cartesian Equation of a Plane

A Normal Equation of a Plane
A plane may be determined by a *point* $P_0(x_0, y_0, z_0)$ and a *vector* perpendicular to the plane $\vec{n}$ called the *normal vector*.



If $P(x, y, z)$ is a generic point on the plane, then:
$\overrightarrow{P_0P} \perp \vec{n}$ and:

$$\overrightarrow{P_0P} \cdot \vec{n} = 0 \quad (1)$$

This is the *normal equation* of a plane.

B Cartesian Equation of a Plane
Let write the normal vector of a plane in the form:

$$\vec{n} = (A, B, C)$$

Then, the normal equation (1) may be written as:
$(x - x_0, y - y_0, z - z_0) \cdot (A, B, C) = 0$
$Ax + By + Cz - Ax_0 - By_0 - Cz_0 = 0$
or:

$$Ax + By + Cz + D = 0 \quad (2)$$

equation which is called the *Cartesian equation* of a plane.

Note. A normal vector to the plane is:

$$\vec{n} = \vec{u} \times \vec{v}$$

where $\vec{u}$ and $\vec{v}$ are the direction vectors of the plane.

Ex 1. Consider the plane π defined the Cartesian equation $\pi : 2x - 3y + 6z + 12 = 0$.

a) Find a normal vector to this plane.

b) Find two points on this plane.

c) Find if the point $P(1,2,3)$ is a point on this plane.

Ex 2. Find the Cartesian equation of a plane π that passes through the points $A(1,-1,0)$, $B(0,0,1)$, and $C(0,-2,1)$.

Ex 3. Find parametric and vector equations for the plane:
$\pi : x - 2y + 3z - 6 = 0$

Ex 4. Find the intersections with the coordinate axes for the plane $\pi : 3x + 2y + z - 6 = 0$. Represent the plane graphically.

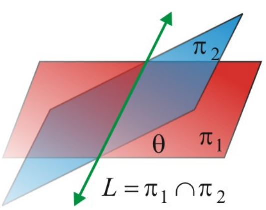
Ex 5. Find the Cartesian equation of a plane with $x-\text{int}=-1$, $y-\text{int}=2$, and $z-\text{int}=-3$.

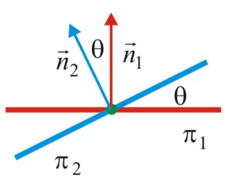
F Angle between two Planes

The *angle* between two plane s is defined as the angle between their *normal vectors*:

$$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

Note. Using this formula, you may get an *acute* or an *obtuse* angle depending on the normal vectors which are used.





Ex 6. Find the angle between each pair of planes.

a) $\pi_1: x+2y+3z+1=0$, $\pi_2: 3x+2y+z+2=0$

b) $\pi_1: x+y+z+1=0$, $\pi_2: x-y-1=0$

Reading: Nelson Textbook, Pages 461-468
Homework: Nelson Textbook: Page 468 #1, 5, 7, 8, 9a, 11, 13, 17

