



Unit 3: Probability

Lesson 3.3: Independent Events and Dependent Events

Part I: Independent Events

If the occurrence of one event has no effect on the occurrence of another, the events are _____.

Product Rule (Fundamental Counting Principle) for independent Events

If A and B are independent events, then the probability of both occurring is given by:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

Example 1:

- a) A coin is tossed 4 times turning up heads each time. What is the probability the fifth trial will be heads?

Explain: Each coin toss is _____ of the others. Although we may think “heads has to come up sometime”, there is still only a half chance on each independent toss. The coin has no memory of the previous four tosses. Therefore, the 5th toss has a _____ probability of getting heads.

- b) Find the probability of tossing five heads in a row.

$$P(H \cap H \cap H \cap H \cap H) = P(H) \times P(H) \times P(H) \times P(H) \times P(H) = \underline{\hspace{10em}}$$

Hence, there is a _____ chance of getting 5 heads in a row.

- c) A coin is flipped while a die is rolled. What is the probability that you will flip a tail and roll a four?

$$P(\text{Tail} \cap 4) = P(\text{Tail}) \times P(4) = \underline{\hspace{10em}}$$

Hence, the probability of tossing tails and rolling a 4 is _____

Example 2: There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that

- a) both tests give the correct result.
- b) neither test gives the correct result
- c) at least one of the tests gives the correct result





Example 3: Pinder has examinations coming up in data management and biology. He estimates that his odds in favour of passing the data-management examination are 17:3 and his odds against passing the biology examination are 3:7. Assume these to be independent events.

- What is the probability that Pinder will pass both exams?
- What are the odds in favour of Pinder failing both exams?

Part II: Dependent events

If the occurrence of one event influences the probability of another event, the events are _____.

When this happens, you can still multiply their probabilities of each event to find the likelihood of A and B (i.e., $P(A \cap B)$). However, you must use the **conditional probability** for the second event.

The conditional probability of B, $P(B|A)$, is the probability that B occurs, given that A has already occurred. The sample space for the second event is reduced by the first event.

Product Rule (Fundamental Counting Principle) for Dependent Events

The probability of dependent events occurring is $P(A \text{ and } B) = P(A) \times P(B|A)$

By rearranging the formula, conditional probability is $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

Example 4:

- Part 1 - Suppose you have a bag containing 2 black marbles and 3 red marbles. You reach into the bag, select a marble, see what color it is and replace it in the bag (Event #1). Then you repeat this process (Event #2). What is the probability of picking a red marble both times?

Solution: Since the first marble was replaced back in the bag before the second marble was drawn, the probability of the second drawing is independent of the probability of the first drawing. These are referred to as independent events --- in other words, the outcome of one event does not affect the outcome of the other event.

- Part 2 - Suppose you have a bag containing 2 black marbles and 3 red marbles. You reach into the bag, select a marble, see what color it is but **DO NOT** replace it in the bag (Event #1). Then you reach in and select another marble (Event #2). What is the probability of picking a red marble both time?

Solution: The probability of picking a red marble the first time (Event #1) is the same as it was in part 1), which is 3 out of 5, or $\left(\frac{3}{5}\right)$. However, since the first marble was not replaced back in the bag, the probability



of picking a red marble the second time is dependent on the outcome of the first drawing. Suppose we did pick a red marble and did not put it back in the bag. Now there are only 2 red marbles and 2 black marbles in the bag, and the probability of picking a red marble the second time (Event #2) is 2 out of 4. Therefore, in this case, Event #1 and Event #2 occurring simultaneously will have a probability of the product of the two dependent event probabilities.

- Hence: **An independent event occurs with replacement. A dependent event occurs without replacement.**

Independent: when the outcome of one event does not change the probability of the other.

Dependent: the probability of the second outcome depends on the results of the first.

Example 5: Tell whether the events are independent or dependent.

- a) You select a card from a deck of 52 cards, replace that card, and select another card.
- b) Ms. Ella chooses students at random to present their projects. She chooses you, and then another student from the remaining students.
- c) There are 10 winning tickets in a collection of 500 tickets. You select a ticket, put it aside, and select another ticket.

Example 6: Lars is offering juice samples at a shopping mall. The experimental probability of a randomly chosen shopper accepting a sample is 15%. The conditional probability of a customer purchasing juice given that they tried a sample is 20%. No one purchases juice without trying a sample. If Lars offers 500 people juice samples, how many sales will he make?

Let A be "try sample", B be "buy juice", and A and B will be _____.

Namely, $P(A) = \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}} = 0.2$

Hence: $P(A \text{ and } B) = P(A) \times P(B|A) =$

Example 7: Rocco and Biff are two koala bears participating in a series of animal behavior tests. They each have 10 mins to solve a maze. Rocco has an 85% probability of succeeding if he can smell the treat. He can smell the treat 60% of the time. Biff has a 70% chance of smelling the treat, but when he does, he can solve the maze only 75% of the time. Neither bear will try to solve the maze unless he smells the treats. Determine which koala bear is more likely to enjoy a tasty treat on any given trial.



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Example 8: Grace is a kicker on her rugby team. She estimates that her chances of scoring on a penalty kick during a game are 75% when there is no wind, but only 60% on a windy day. If the weather forecast gives a 55% chance of windy weather this afternoon, what is the probability of Grace scoring on a penalty kick in this afternoon's game?



Example 9: The table shows the status of 200 registered college students. A student is randomly selected.

- What is the probability that the selected student is female?
- What is the probability that a selected female student is a full time student?

	Part time	Full time
Female	80	40
Male	60	20

$P(B|A)$ is the probability of a selected female is full time student;

$P(A \text{ and } B)$ is the probability of a student is both a female and a full time student;

$P(A)$ is the probability of a selected student is a female.



Practise

A

1. Classify each of the following as independent or dependent events.

	First Event	Second Event
a)	Attending a rock concert on Tuesday night	Passing a final examination the following Wednesday morning
b)	Eating chocolate	Winning at checkers
c)	Having blue eyes	Having poor hearing
d)	Attending an employee training session	Improving personal productivity
e)	Graduating from university	Running a marathon
f)	Going to a mall	Purchasing a new shirt

2. Amitesh estimates that he has a 70% chance of making the basketball team and a 20% chance of having failed his last geometry quiz. He defines a “really bad day” as one in which he gets cut from the team and fails his quiz. Assuming that Amitesh will receive both pieces of news tomorrow, how likely is it that he will have a really bad day?
3. In the popular dice game Yahtzee®, a Yahtzee occurs when five identical numbers turn up on a set of five standard dice. What is the probability of rolling a Yahtzee on one roll of the five dice?

Apply, Solve, Communicate

B

4. There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that
- both tests give the correct result
 - neither test gives the correct result
 - at least one of the tests gives the correct result

5. a) Rocco and Biff are two koala bears participating in a series of animal behaviour tests. They each have 10 min to solve a maze. Rocco has an 85% probability of succeeding if he can smell the eucalyptus treat at the other end. He can smell the treat 60% of the time. Biff has a 70% chance of smelling the treat, but when he does, he can solve the maze only 75% of the time. Neither bear will try to solve the maze unless he smells the eucalyptus. Determine which koala bear is more likely to enjoy a tasty treat on any given trial.

- b) **Communication** Explain how you arrived at your conclusion.
6. Shy Tenzin’s friends assure him that if he asks Mikala out on a date, there is an 85% chance that she will say yes. If there is a 60% chance that Tenzin will summon the courage to ask Mikala out to the dance next week, what are the odds that they will be seen at the dance together?
7. When Ume’s hockey team uses a “rocket launch” breakout, she has a 55% likelihood of receiving a cross-ice pass while at full speed. When she receives such a pass, the probability of getting her slapshot away is $\frac{1}{3}$. Ume’s slapshot scores 22% of the time. What is the probability of Ume scoring with her slapshot when her team tries a rocket launch?

8. **Inquiry/Problem Solving** Show that if A and B are dependent events, then the conditional probability $P(A | B)$ is given by

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$$



9. A consultant's study found Megatran's call centre had a 5% chance of transferring a call about schedules to the lost articles department by mistake. The same study shows that, 1% of the time, customers calling for schedules have to wait on hold, only to discover that they have been mistakenly transferred to the lost articles department. What are the chances that a customer transferred to lost articles will be put on hold?
10. Pinder has examinations coming up in data management and biology. He estimates that his odds in favour of passing the data-management examination are 17:3 and his odds against passing the biology examination are 3:7. Assume these to be independent events.
- What is the probability that Pinder will pass both exams?
 - What are the odds in favour of Pinder failing both exams?
 - What factors could make these two events dependent?
11. **Inquiry/Problem Solving** How likely is it for a group of five friends to have the same birth month? State any assumptions you make for your calculation.
12. Determine the probability that a captured deer has the bald patch condition.



13. **Communication** Five different CD-ROM games, Garble, Trapster, Zoom!, Bungie, and Blast 'Em, are offered as a promotion by SugarRush cereals. One game is randomly included with each box of cereal.
- Determine the probability of getting all 5 games if 12 boxes are purchased.
 - Explain the steps in your solution.
 - Discuss any assumptions that you make in your analysis.

14. **Application** A critical circuit in a communication network relies on a set of eight identical relays. If any one of the relays fails, it will disrupt the entire network. The design engineer must ensure a 90% probability that the network will not fail over a five-year period. What is the maximum tolerable probability of failure for each relay?



15. a) Show that if a coin is tossed n times, the probability of tossing n heads is given by $P(A) = \left(\frac{1}{2}\right)^n$.
- b) What is the probability of getting at least one tail in seven tosses?
16. What is the probability of not throwing 7 or doubles for six consecutive throws with a pair of dice?
17. Laurie, an avid golfer, gives herself a 70% chance of breaking par (scoring less than 72 on a round of 18 holes) if the weather is calm, but only a 15% chance of breaking par on windy days. The weather forecast gives a 40% probability of high winds tomorrow. What is the likelihood that Laurie will break par tomorrow, assuming that she plays one round of golf?
18. **Application** The Tigers are leading the Storm one game to none in a best-of-five playoff series. After a playoff win, the probability of the Tigers winning the next game is 60%, while after a loss, their probability of winning the next game drops by 5%. The first team to win three games takes the series. Assume there are no ties. What is the probability of the Storm coming back to win the series?