Probability Questions

1. A jar contains four marbles: three red, one white. Two marbles are drawn with replacement. (i.e. A marble is randomly selected, the color noted, the marble replaced in the jar, then a second marble is drawn.)

   a. List a sample space containing four outcomes.
   b. List a sample space with sixteen outcomes.
   c. Write the probability of each of the four outcomes in (a).
   d. What are the probabilities of the outcomes in (b)?
   e. What is the probability the colors of the two marbles match?
   f. What is the probability the same marble is drawn twice?

2. We are playing with a short deck, as shown at right.
   Let "H" be the event the card drawn is a heart.
   Let "D" be the event the card drawn is a diamond.
   Let "A" be the event the card is an ace.

A♥	A♦	A♣	A♠
2♥	2♦	2♣	2♠
3♥	3♦	3♣	3♠
4♥	4♦	4♣	4♠

   a. P(H) =   P(D) =   P(A) =
   b. P(H or D) =   c. P(H or A) =
   d. P(H and D) =   e. P(H and A) =
   f. Are H and D *independent events*?   g. Are H and A *independent events*?

3. If three cards are drawn from the deck in #2, one at a time, what is the probability that
   a. the 1st card is the ace of hearts, the 2nd is the 2 of diamonds, and the 3rd is the 3 of clubs?
   b. all three cards are aces?

4. An airplane is built to be able to fly on one engine. If the plane's two engines operate independently, and each has a 1% chance of failing in any given four-hour flight, what is the chance the plane will fail to complete a four-hour flight to Oklahoma due to engine failure?

5. A pair of fair, standard dice are rolled. What is the probability the sum of the dice is 5?

6. Fifty marbles are to be drawn from the jar in problem #1 with replacement. If the first four marbles drawn are red, what is the probability the next marble drawn will *not* be red?

7. A probability experiment has four possible outcomes: e_1, e_2, e_3, e_4. The outcome e_1 is four times as likely as each of the three remaining outcomes. Find the probability of e_1.

8. What are the odds in favor of rolling a sum of seven in one roll of a pair of fair standard dice?

9. If P(A) = ½ and P(B) = ½ and P(B|A) = 1/3, find:

   a. P(A and B)
   b. P(A or B)
   c. P(A| B)

10. The deck of sixteen cards shown in #2 is thoroughly shuffled. Three cards are drawn from the top of the deck, one at a time. What is the probability the third card is an ace?
    (Hint: There is a really simple, direct solution.)

11. "The Birthday Problem" (famous) In a roomful of 30 people, what is the probability that at least two people have the same birthday? Assume birthdays are uniformly distributed and there is no leap year complication. *(Hint: what is the probability that they all have different birthdays?)*

12. A 1-inch-diameter coin is thrown on a table covered with a grid of lines two inches apart. What is the probability the coin lands *in* a square *without touching any of the lines of the grid*?
    (Hint: in order that the coin not touch any of the grid lines, where must the center of the coin be?)

Too-Hard Probability Answers:

1a. {RR, RW, WR, WW}

1c. 9/16 3/16 3/16 1/16 respectively

1b. { R_1R_1, R_1R_2, R_1R_3, R_1W_1
R_2R_1, R_2R_2, R_2R_3, R_2W_1
R_3R_1, R_3R_2, R_3R_3, R_3W_1
W_1R_1, W_1R_2, W_1R_3, W_1W_1 }

1d. The outcomes detailed in the sample space in *1b* are equally likely; each has *P* = 1/16.

1e. P(colors match) = P(RR) + P(WW) = 9/16 + 1/16 = 10/16 or 5/8 $\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{10}{16}=\frac{5}{8}$

1f. P(same marble twice) = P(R_1R_1,R_2R_2,R_3R_3,W_1W_1} = 4/16 (using *1b*; SS in *1a* is no helpat all)

...or, you can reason thus: P(same marble twice) = P(second marble is same as the first) = 1/4 because there are 4 marbles in the jar on the second draw, and only one is the same marble as the 1^{st}.

2a. P(H) = P(♥) = P(A♥, 2♥, 3♥, 4♥} = 4/16 ...or... P(♥) = P(♥, 1 of the 4 equally likely suits) = 1/4
P(D) = P(♦) = P(♥) = 1/4
P(A) = P({A♥, A♦, A♣, A♠}) = 4/16 = 1/4

2b. P(H or D) = P(H) + P(D) *because the events*
1/4 + 1/4 = 1/2 *H and D are disjoint.*

2c. P(H or A) = P(H) + P(A) – P(H and A) =
1/4 + 1/4 – 1/16 = 7/16

2d. P(H and D) = 0 *(see 2b)*

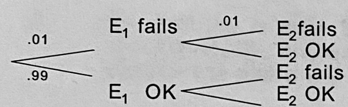
2e. P(H & A) = P(A♥) = 1/16

2f. H & D are **not** independent, they are *mutually exclusive*. If one occurs, the other cannot!

2g. P(A♥) = P(A)·P(♥) ...so: yes, they are *independent*.
Also, P(♥) = 4/16 = 1/4 = P(♥| A). ♥ has same P if A.

3. a. P(A♥) P(2♦ | A♥ gone) P(3♣ | 2♦&A♥ gone) = (1/16) (1/15) (1/14)
b. P(AAA) = (4/16) (3/15) (2/14) *...by reasoning similar to part a.*

4.



The plane will fail to make the flight due to engine failure *only* if BOTH engines fail (because the plane can fly on one engine.)
P(flight fails) =
P(BOTH engines fail) = P(1st fails)·P(2nd fails) = .0001

5. P(*sum* = 5) = *P(rolling 14 or 23 or 32 or 41) = 4/36 = 1/9*

6. Every time a marble is taken from this jar (assuming previously drawn marbles are replaced), the probability of obtaining a red marble is 3/4. Therefore, P(not red) = 1/4.

7. $4p + p + p + p = 1 \Rightarrow 7p = 1 \Rightarrow p = 1/7. \Rightarrow P(e_1) = 4p = 4(1/7) = 4/7$

8. There are six ways to roll a sum of 7: 16, 25, 34, 43, 52, 61. P(sum = 7) = 6/36 or 1/6 (not the question!)

There are six favorable outcomes in this SS with 36 equally likely outcomes, so 29 are unfavorable.
The odds in favor of a sum of 7 are 6:29
(Because they are 29:6 against....)

9. a. P(A and B) = P(A) P(B| A) = (½) (⅓) = $^1/_6$

b. P(A or B) = P(A) + P(B) – P(A and B) = ½ + ½ – $^1/_6$ = $^5/_6$

c. $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{^1/_6}{½} = ⅓$

We note that A & B are NOT independent. P(A| B) ≠ P(A) *(showing B has an effect on A!)*
Also P(A and B) = P(A) P(B| A) = $^1/_6$ ≠ ¼ = P(A)•P(B)

11. It is difficult to calculate directly the chance of at least two matching birthdays, because you have to allow for so many possibilities: just two matching, three matching, two pairs matching, etc. etc. The COMPLEMENT of this event is, however, quite simple. If there are NOT at least two matching birthdays, then there are NONE!

P(all different) = $\frac{365}{365}\frac{364}{365}\frac{363}{365}\cdots\frac{336}{365}$ (Here it is appropriate to use a calculator– carefully.) This turns out to be under 30%.

Therefore, the probability that at least two birthdays match is over 70%!

12. *Where does the coin have to land in order to win? What determines the location of the coin? Where must the center of the coin be? Draw a picture of where it can be. The answer is one-fourth.*