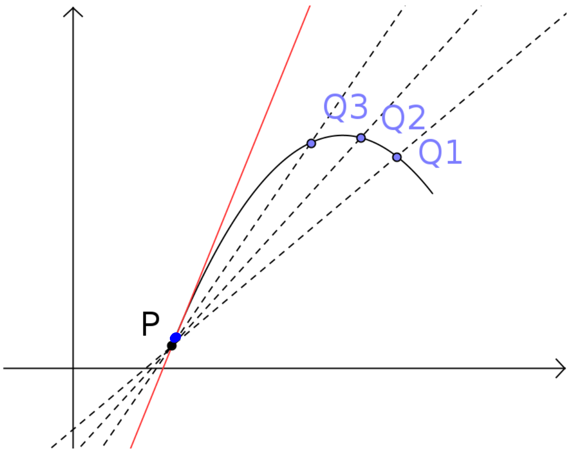
Slope of a Tangent (as a limit)



The slope of the tangent line at P can be approximated by the slope of the secant line between P and Q.

As Q gets closer to P, the slope of the secant will approach the slope of the tangent.

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Recall: Rate of Change

$x = a$

$$\boxed{m_{\text{secant}} = \frac{f(a+h) - f(a)}{h}}$$

Smaller values of 'h' result in a better estimate for the slope of the tangent.

As 'h' approaches zero, the slope of the secant approaches the slope of the tangent.

$$\boxed{m_{\text{tangent}} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

"the limit as h approaches zero"

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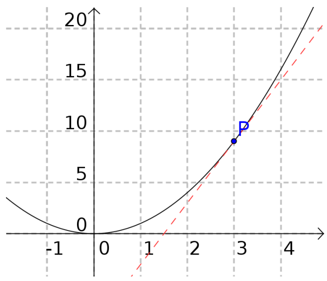
Ex. Determine the slope of the tangent to the given curve when x = 3. $\rightarrow a = 3$

(a) $y = x^2$

$f(x) = x^2$



$$m_{tangent} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$m = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} \frac{(3+h)^2-(3)^2}{h}$$

$$= \lim_{h\to 0} \frac{(9+6h+h^2)-9}{h}$$

$$= \lim_{h\to 0} \frac{6h+h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} \qquad h \neq 0$$

$$= \lim_{h\to 0} 6+h$$

$$= 6 \checkmark$$

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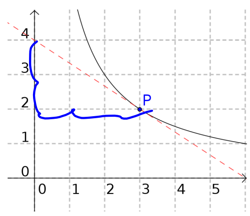
Ex. Determine the slope of the tangent to the given curve when x = 3.

Whenever possible, change the expression algebraically to remove 'h' from any denominators, allowing the limit to be evaluated exactly.

(b) $y = \frac{6}{x}$

$f(x) = \frac{6}{x}$



$$m = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6}{3+h}-\frac{6}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6}{3+h}-2}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6-2(3+h)}{3+h}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6-6-2h}{3+h}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2h}{3+h}}{h}$$

$$= \lim_{h\to 0} \left(\frac{-2\cancel{h}}{3+h}\right)\left(\frac{1}{\cancel{h}}\right)$$

$$= \lim_{h\to 0} \frac{-2}{3+h}$$

$$= \frac{-2}{3} \checkmark$$

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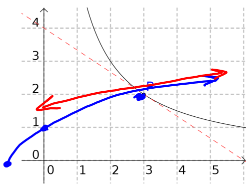
Ex. Determine the slope of the tangent to the given curve when x = 3.

(c) $y = \sqrt{x+1}$



$$f(x) = \sqrt{x+1}$$

$$m = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{(3+h)+1} - \sqrt{3+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h} \times \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$= \lim_{h\to 0} \frac{(\sqrt{4+h})^2 - (2)^2}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{(4+h) - 4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}^{1}}{\cancel{h}_{1}(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2}$$

$$= \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

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Assigned Work:

p.19 # 4, 5 (basics)
p.20 # 8c, 9ac, 10b, 11ef, 16, 20, 25*

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p. 20 # 25.

(a) $y = 4x^2 + 5x - 2$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[4(a+h)^2 + 5(a+h) - 2] - [4a^2 + 5a - 2]}{h}$$

$$= \lim_{h \to 0} \frac{[4(a^2 + 2ah + h^2) + 5a + 5h - 2] - [4a^2 + 5a - 2]}{h}$$

$$= \lim_{h \to 0} \frac{[\cancel{4a^2} + 8ah + 4h^2 + \cancel{5a} + 5h - \cancel{2}] - [\cancel{4a^2} + \cancel{5a} - \cancel{2}]}{h}$$

$$= \lim_{h \to 0} \frac{4h^2 + 8ah + 5h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4h + 8a + 5)}{\cancel{h}}$$

$$= 8a + 5$$

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25. (a) $m_T = 8a + 5$

(b) parallel to $10x - 2y - 18 = 0$

$$\frac{10x - 18}{2} = \frac{2y}{2}$$

$$y = 5x - 9$$

$m_{//} = 5$

Set $m_T = m_{//}$

$5 = 8a + 5$

$0 = 8a$

$a = 0$

(c) $m_{\perp} = \frac{-1}{m}$

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