Chapter 3: Derivatives and their Applications

3.2 Maximum and Minimum on an Interval

Learning Goal:

You will understand that a max/min
can occur at either a peak/valley, or at the
end of an interval

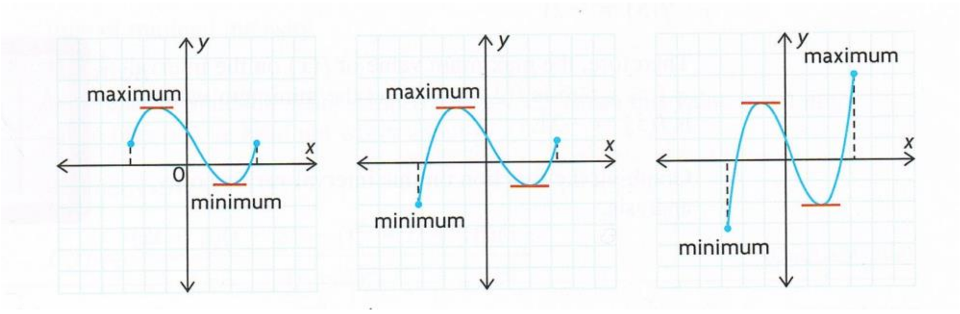
You will use derivatives to find the max or min

3.2 Maximum and Minimum on an Interval

- also called the extreme values, or absolute extrema
- the highest or lowest values within a specified interval

For a function that has a derivative at every point in an interval

the maximum value occurs at a "peak" **or** at the endpoint of the interval
the minimum value occurs at a "valley" **or** at the endpoint of the interval.



To find the extrema for f(x) in the interval $a \le x \le b$

- Find all points at which $f'(x) = 0$ (this gives any peaks or valleys).
- Determine f(x) (ie the "y") for these points, and also for the endpoints.
- Compare the values: the largest is the maximum value and the smallest is the minimum value

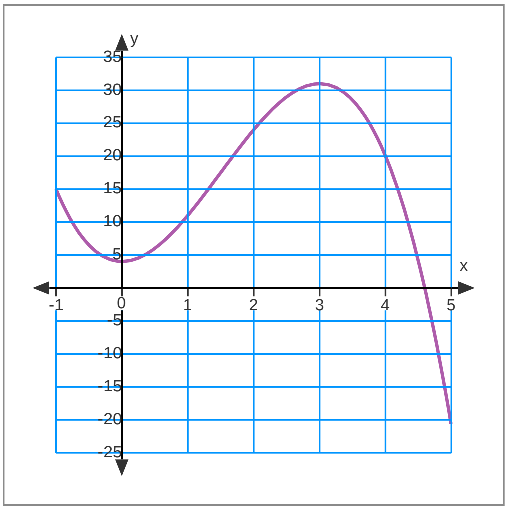
Ex find the extreme values of $f(x) = -2x^3 + 9x^2 + 4$ on the interval $x \varepsilon [-1 , 5]$

$f'(x) = -6x^2 + 18x$

$0 = -6x(x - 3)$ $x = 0$ and 3.

$f(-1) = 15$
$f(0) = 4$
$f(3) = 31$
$f(5) = - 21$

The maximum occurs at $x = 3$ $(3, 31)$
The minimum occurs at $x = 5$ $(5, -21)$



Ex 2. The amount of light intensity on a point is given by the function

$$l(t) = \frac{t^2 + 2t + 16}{t + 2}$$

where t is time in seconds and t $\in$ [0, 14].
Determine the time of minimal intensity.

$$l'(t) = \frac{(2t + 2)(t + 2) - (t^2 + 2t + 16)(1)}{(t + 2)^2}$$

$$l'(t) = \frac{t^2 + 4t - 12}{(t + 2)^2}$$

To set l'(t) = 0, we need only worry about the numerator:

$0 = t^2 + 4t - 12$ $\quad$ $0 = (t - 2)(t + 6)$ $\quad$ t = 2 and ~~-6~~

l(0) = 8
l(2) = 6
l(14) = 15

The minimum light intensity occurs at t = 2.

Homework:

p. 135 # 1ad, 2, 3acf, 4abde, 6, 8, 11, 14