

3.3– Optimization Problems

Goal: To use derivatives to solve optimization problems.

Calculus can be used to find critical points which are usually the maximum and minimum values of a function. This simple concept has countless applications. Consider a profit function, $P(x)$, which companies would want to MAXIMIZE, or a cost function, $C(x)$, which companies would want to MINIMIZE. Finding this “best”, or optimal, value is called **optimization**.

Steps to solve Optimization Problems:

1. Identify what the question is asking. Draw a diagram, if possible.
2. Define the variable(s).
3. Identify the quantity to be optimized and write an equation.
4. Choose **ONE** independent variable.
5. Define the function-to-be-optimized in terms of only your independent variable.
NOTE: This often involves creating a second equation you can use to eliminate a variable.
6. Differentiate the function-to-be-optimized.
7. Determine the critical points (max / min / saddle point) by setting the derivative equal to zero.
- 8. Answer the question posed in the problem.**

Example 1. We are trying to build two vegetable gardens that need to be fenced off from the animals. The two areas will share one common side and will be built with 60 m of fencing total. One garden will be a square, and the other a rectangle. Find the dimensions that will maximize the total area.

Example 2. We are trying to create a ‘can of worms’ prank for April Fool’s day. The cylindrical can must have a volume of 1 L to provide the most realistic experience. Find the height and radius of the can that will minimize the surface area.

Example 3. For an outdoor concert, a ticket price of \$30 typically attracts 5000 people. For each \$1 increase in the ticket price, 100 fewer people will attend. The revenue, R , is the product of the number of people attending and the price per ticket. Find the ticket price that maximizes revenue.

Example 4. A cardboard box with a square base is to have a volume of 8 L. The cardboard for the box costs 0.1 cents/m², but the cardboard for the bottom is thicker, so its costs three times as much. Find the dimensions that will minimize the cost of the cardboard.

IP: P. 201 #4a, 9, 10b, 12 **Note:** Volume and surface area formulas are on the last page of textbook.