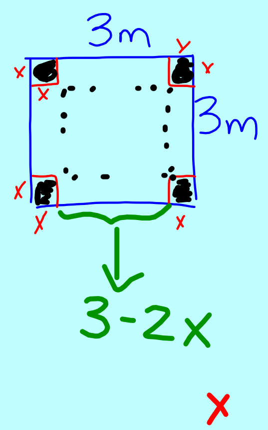
Lesson 3: Optimization Problems

Example 1: (Polynomial)-See Lesson 3 computer
A box with an open top is to be constructed from a square piece of cardboard, 3m wide, by cutting out a square piece from each of the four corners and bending up the sides. Find the largest volume of such a box.



Visual

$$V = lwh$$
$$= (3-2x)(3-2x)(x)$$
$$= 9x - 12x^2 + 4x^3$$
$$V'(x) = 9 - 24x + 12x^2$$
$$12x^2 - 24x + 9 = 0$$
$$(2x-1)(2x-3) = 0$$
$$x = \frac{1}{2}, \cancel{\frac{3}{2}} \text{ too BIG}$$
$$V\left(\frac{1}{2}\right) = \left(3 - 2\left(\frac{1}{2}\right)\right)^2\left(\frac{1}{2}\right)$$
$$= 2m^3$$

Example 2: (Polynomial)
A string of length 50 m is cut in two pieces. One piece is bent into a circle and the other is bent into an equilateral triangle. Find an equation for the total area of the two shapes, and where should the string be cut to minimize the area?

Visual

x | $50-x$

50m

○ $\Rightarrow A = \pi r^2$

△ $\Rightarrow A = \frac{b \times h}{2}$

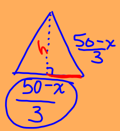
○ $\Rightarrow C = 2\pi r$

$x = 2\pi r$

$\boxed{\frac{x}{2\pi} = r}$

△ $\Rightarrow P = 50-x$

$1\text{side} = \frac{50-x}{3}$



$$h = \sqrt{\left(\frac{50-x}{3}\right)^2 - \left(\frac{50-x}{6}\right)^2}$$

$$= \boxed{\sqrt{\frac{2500-100x+x^2}{9} - \frac{(2500-100x+x^2)}{36}}}$$

$$A = \pi\left(\frac{x}{2\pi}\right)^2 + \frac{\left(\frac{50-x}{3}\right)\left(\sqrt{\frac{2500-100x+x^2}{9} - \frac{(2500-100x+x^2)}{36}}\right)}{2}$$

$$= \frac{x^2}{4\pi} + \frac{1}{2}\left[\left(\frac{50}{3} - \frac{x}{3}\right)\left(\quad\right)\right]$$

$$A' = \frac{2x}{4\pi} + \frac{1}{2}\left[-\frac{1}{3}\left(\sqrt{\frac{2500-100x+x^2}{9} - \frac{(2500-100x+x^2)}{36}}\right)\right.$$

$$\left. + \left(\frac{50}{3} - \frac{x}{3}\right)\left(\frac{1}{2}\left(\quad\right)^{-\frac{1}{2}}\left(-\frac{100}{9} + \frac{2x}{9} + \frac{100}{36} - \frac{2x}{36}\right)\right)\right]$$

$$\boxed{A_{TOTAL} = \frac{\sqrt{3}}{x}x^2 + \frac{1}{4\pi}(50-x)^2}$$

3.12m / 1.88m

Example 3: (Economics)
A manufacturer of calculators produces x calculators per day at a daily cost in dollars of $\boxed{C(x) = 40x - 0.035x^2 + 1250.}$ If the calculators are sold for 60 - 0.05x each, find the value of x that maximizes the daily profit. What is the price for maximum profit?

$P = R - C$

Price

$P = (60 - 0.05x)x - (40x - 0.035x^2 + 1250)$

$= 60x - 0.05x^2 - 40x + 0.035x^2 - 1250$

$= -0.015x^2 + 20x - 1250$

$P' = -0.03x + 20$

$\frac{-\cancel{0.03}x}{-\cancel{0.03}} = \frac{-20}{-0.03}$

$x \doteq 666.7$ units to sell

$\therefore$ Price $= 60 - 0.05(667)$

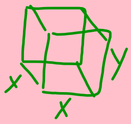
$= \$26.65$

Example 4: (Rational)
An open topped storage box with a square base is to have a capacity of 5m³. Material for the sides costs \$1.60/m², while that for the bottom costs \$2.00/m². Find the dimensions that will minimize the cost of the material. What is the minimum cost?

$V = 5m^3$

$l = w = x$

$h = y$



$V = l \times w \times h$

$5 = x^2 y$

$\frac{5}{x^2} = y$

$1 \text{ side} = xy$

$4 \text{ sides} = 4xy$

$\text{bottom} = x^2$

$$C = 1.6(4xy) + 2x^2$$

$$C(x) = 1.6\left(4x\left(\frac{5}{x^2}\right)\right) + 2x^2$$

$$= \frac{32}{x} + 2x^2$$

$$C'(x) = -\frac{32}{x^2} + 4x$$

$$\frac{32}{x^2} = 4x$$

$$\frac{4x^3}{4} = \frac{32}{4}$$

$$x^3 = 8$$

$$x = 2$$

dimensions:

length $= x = 2m$

width $= x = 2m$

height $= y = \frac{5}{(2)^2} = 1.25m$

min. cost $= C(2) = \frac{32}{2} + 2(2)^2$

$= 16 + 8$

$= \$24$

Attachments

Ex2TriangleCircle.gsp