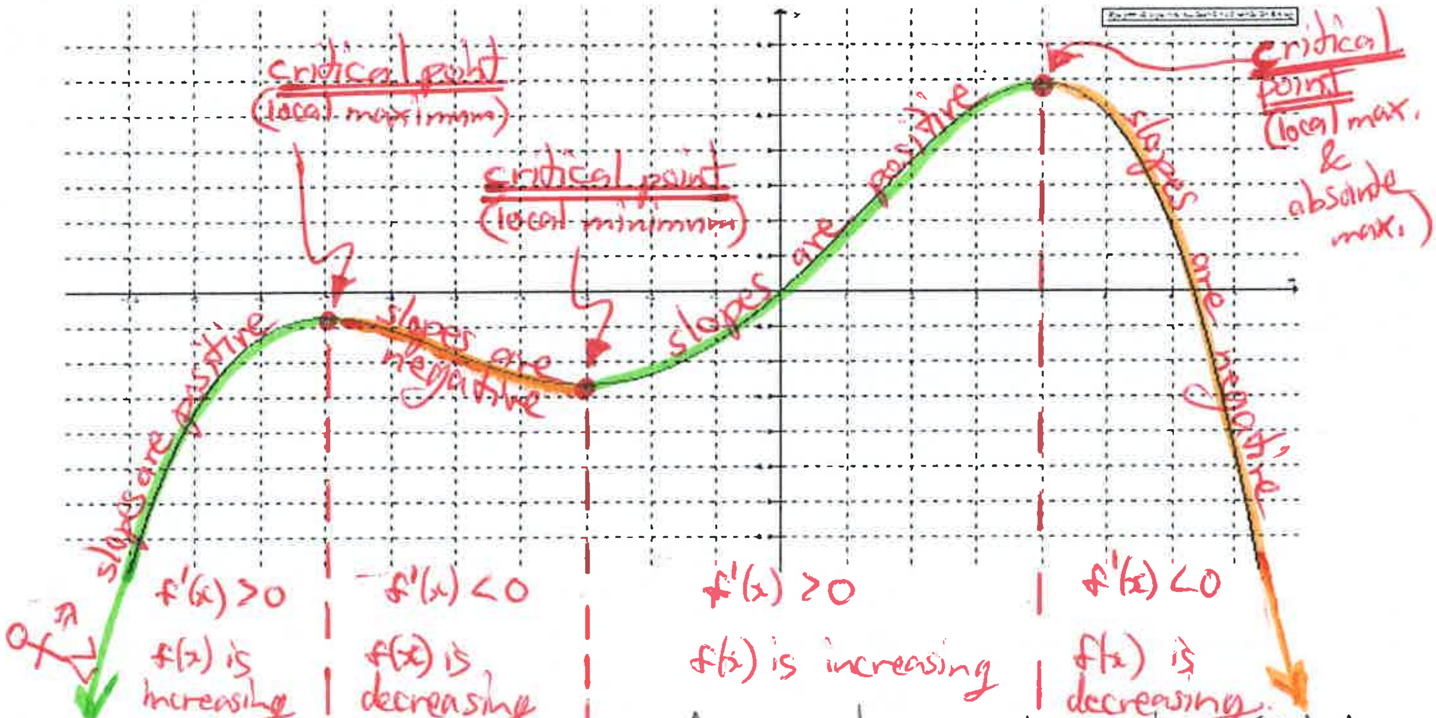
Date: COMPLETED.

3.1 Increasing and Decreasing Functions

Goal: To define and identify increasing/decreasing functions and critical points, perform a first derivative test, and to use the first derivative/properties to sketch a function.



- A function is **INCREASING** on an interval if the y-values are increasing (rising)

  The slope of the tangent will be positive (first derivative is positive).

$f(x)$ is increasing if $f'(x)>0$ ⊕

- A function is **DECREASING** on an interval if the y-values are decreasing (falling)

  The slope of the tangent will be negative (first derivative is negative).

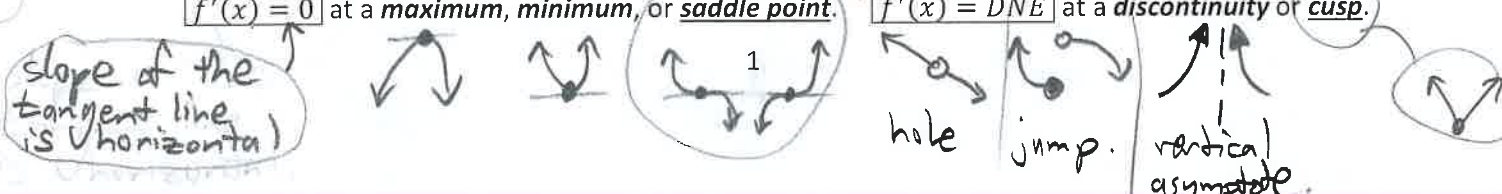
$f(x)$ is decreasing if $f'(x)<0$ ⊖

- A function is at a **CRITICAL POINT** if it is neither increasing nor decreasing

  The slope of the tangent is neither positive nor negative (zero or undefined)

$f(x)$ is a critical point if $f'(x)=0$ or $f'(x)=\emptyset$ or DNE

$\boxed{f'(x)=0}$ at a ***maximum***, ***minimum***, or ***saddle point***. $\boxed{f'(x)=DNE}$ at a ***discontinuity*** or ***cusp***.



Example 1: Determine values of x for which the derivative of $f(x) = \frac{1}{4}x^4 - 2x^3$ equals zero.

$f'(x) = x^3 - 6x^2$

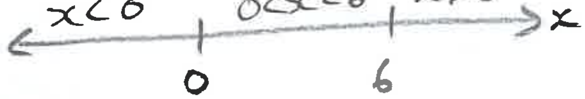
$0 = x^2(x-6)$

$x^2 = 0 \qquad x - 6 = 0$

$\boxed{x = 0 \qquad x = 6.}$ ⇐ critical numbers ($f'(x) = 0$)

Intervals:

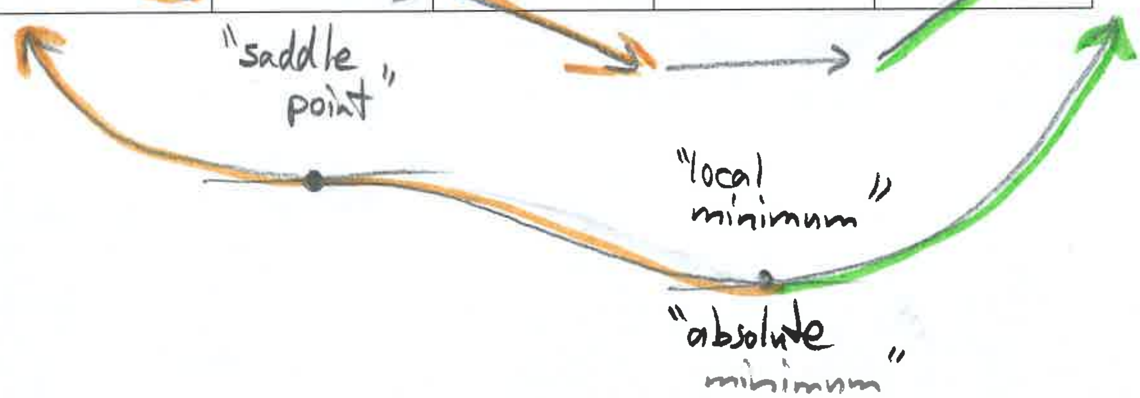
$(-\infty, 0)$ $x < 0$ $\quad (0, 6)$ $0 < x < 6$ $\quad (6, \infty)$ $x > 6$



To find the ***intervals of increase or decrease*** for the function, use the **FIRST DERIVATIVE TEST.**

The ***intervals*** are separated by the ***critical points*** (where $f'(x) = 0$ or $f'(x) = DNE$).

Interval	$(-\infty, 0)$ $x < 0$	$x = 0$	$(0, 6)$ $0 < x < 6$	$x = 6$	$(6, \infty)$ $x > 6$
Test Value	-1	0	1	6	7
Sign of $f'(x)$ [$f'(x) = (x^2)(x-6)$]	$(+)(-) = (-)$	0	$(+)(-) = (-)$	0	$(+)(+) = (+)$
Description of $f(x)$	decreasing	critical point	decreasing	critical point	increasing

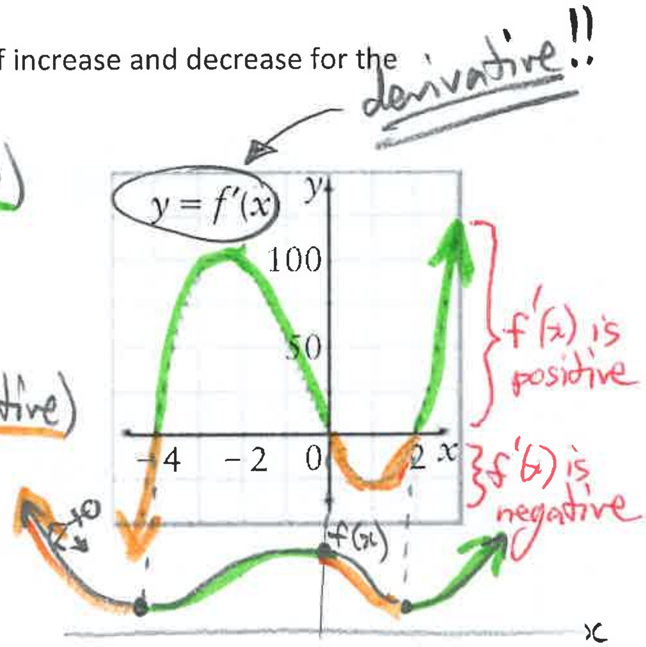


The function is increasing over the interval: $x > 6$

The function is decreasing over the intervals $x < 0$ and $0 < x < 6$

(Note: it is slightly wrong to say decreasing on $x < 6$, because $f(x)$ is not decreasing at the critical point)

Example 2: Given the graph of $f'(x)$ state the intervals of increase and decrease for the function $f(x)$. Sketch a possible graph of $y = f(x)$.



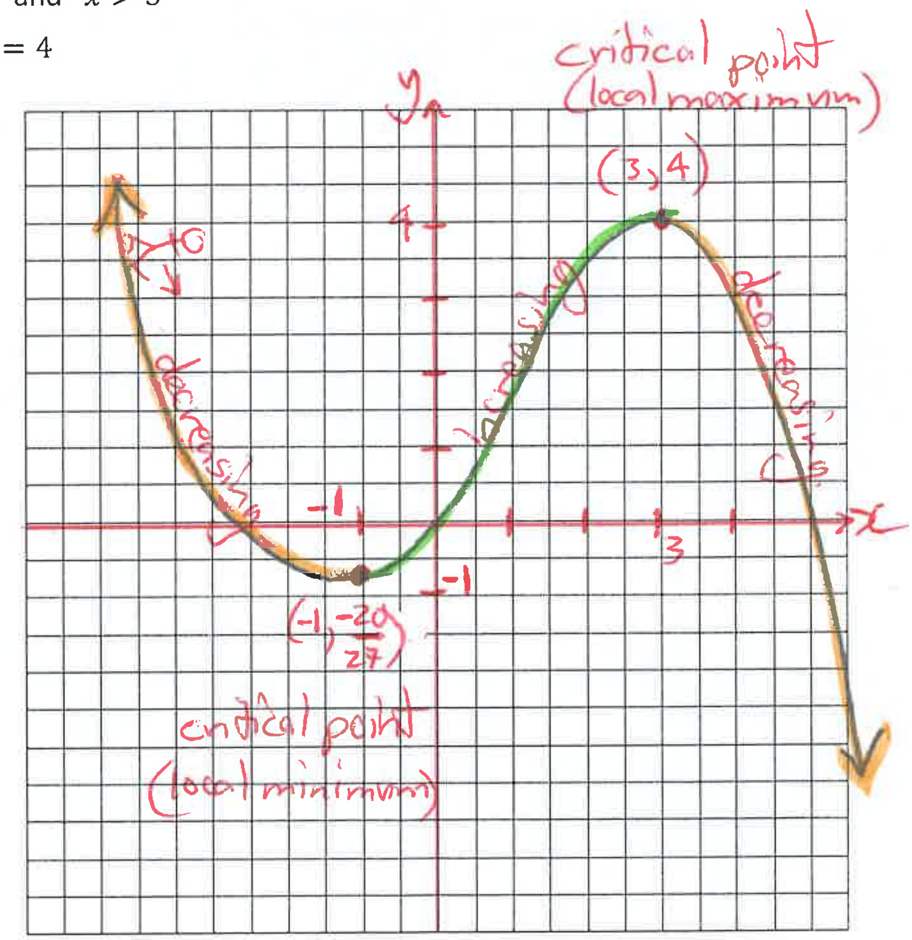
- $f(x)$ is increasing when $f'(x) > 0$ (positive)
  ↳ $\boxed{-4 < x < 0}$ and $\boxed{x > 2}$
- $f(x)$ is decreasing when $f'(x) < 0$ (negative)
  ↳ $\boxed{x < -4}$ and $\boxed{0 < x < 2}$
- Note: critical points at $x = -4, 0, 2$.

Example 3: Sketch a continuous graph that satisfies the set of conditions:

1. $f'(x) > 0$ when $-1 < x < 3$
2. $f'(x) < 0$ when $x < -1$ and $x > 3$
3. $f(-1) = -\frac{20}{27}$ and $f(3) = 4$

$\left(-1, -\frac{20}{27}\right)$ $(3, 4)$

Easiest to graph your known points first!

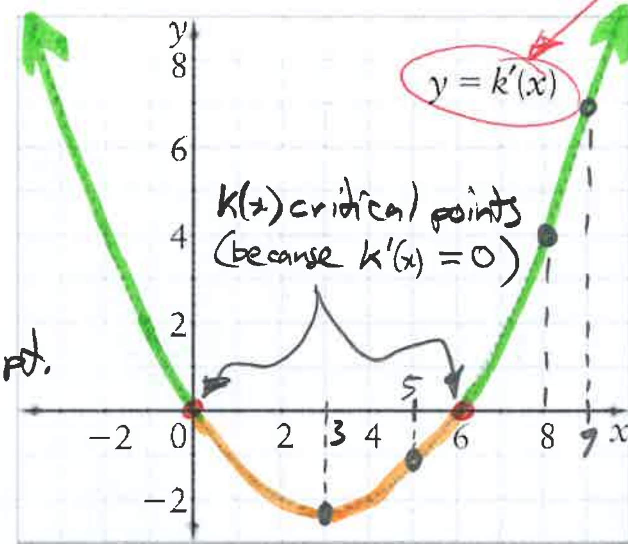


b/c = because

derivative!

Example 4: Given the graph of $k'(x)$, determine which value of x in each pair gives the greater value of $k(x)$. Explain your reasoning.

derivative!



K(x) increasing b/c $k'(x) > 0$

K(x) decreasing b/c $K'(x) < 0$

a) $k(3)$ or $k(5)$

→ decreasing at both points b/c $f'(x) < 0$ (negative)

∴ decreasing on interval

∴ start point is greater than end pt.

$\boxed{k(3) > k(5)}$

b) $k(8)$ or $k(12)$

→ increasing at both points b/c $f'(x) > 0$ (positive)

∴ increasing on the interval

∴ end point is greater than start point

$\boxed{K(12) > K(8)}$

c) $k(9)$ or $k(5)$

→ decreasing at $x=5$ because $f'(x) < 0$ (negative) (a little)

→ increasing at $x=9$ because $f'(x) > 0$ (positive) (a lot)

∴ mostly increasing on the interval (decreasing a little; increasing a lot)

∴ end point is greater than start point

$\boxed{K(9) > K(5)}$

IP - P. 156 #2abce, 4a-f, 6ac, 7bcd