Critical Points & 1st Derivative Test

If f '(a) = 0, then x = a is a critical value and (a, f(a)) is a critical point on f(x).

x - only x y

A critical point can be
- a maximum (slope changes from + to -)
- a minimum (slope changes from - to +)
- a point of inflection (trend in slope continues)

f goes from INC to DEC

PoI

If f '(a) is undefined, then x = a is a critical value .

On f(x), this critical value can occur at
- a vertical asymptote (no critical point, a VA for f(x))
- a max or a min (the critical point is a cusp)

Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.
a) Find all intercepts.
b) Find all critical points.

(a) y-int : $f(0) = 0^4 - 3(0)^3$
$= 0$
$(0,0)$

x-int : $f(x) = 0$
$x^4 - 3x^3 = 0$
$x^3(x-3) = 0$
$x = 0$ or $x = 3$
triple root linear root

(b) $f'(x) = 4x^3 - 9x^2$
set $f'(x) = 0$
$0 = x^2(4x-9)$
$x = 0$ or $4x - 9 = 0$
$x = \frac{9}{4}$

critical values

$f(0) = 0$

$CP_1(0,0)$

$f\left(\frac{9}{4}\right) = \left(\frac{9}{4}\right)^4 - 3\left(\frac{9}{4}\right)^3$
$= \frac{6561}{256} - 3\left(\frac{729}{64}\right)$
$= \frac{-2187}{256}$

$CP_2\left(\frac{9}{4}, \frac{-2187}{256}\right)$

Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.

c) Classify the critical points using an INC/DEC table.

$$f'(x) = 4x^3 - 9x^2$$
$$0 = x^2(4x - 9)$$

-1 0 1 $\frac{9}{4}$ 5

interval	$(-\infty, 0)$	$(0, \frac{9}{4})$	$(\frac{9}{4}, \infty)$
x^2	+	+	+
$4x - 9$	−	−	+
sign of $f'(x)$	−	−	+
INC/DEC?	DEC ↘	DEC ↘	INC ↗

trend continues

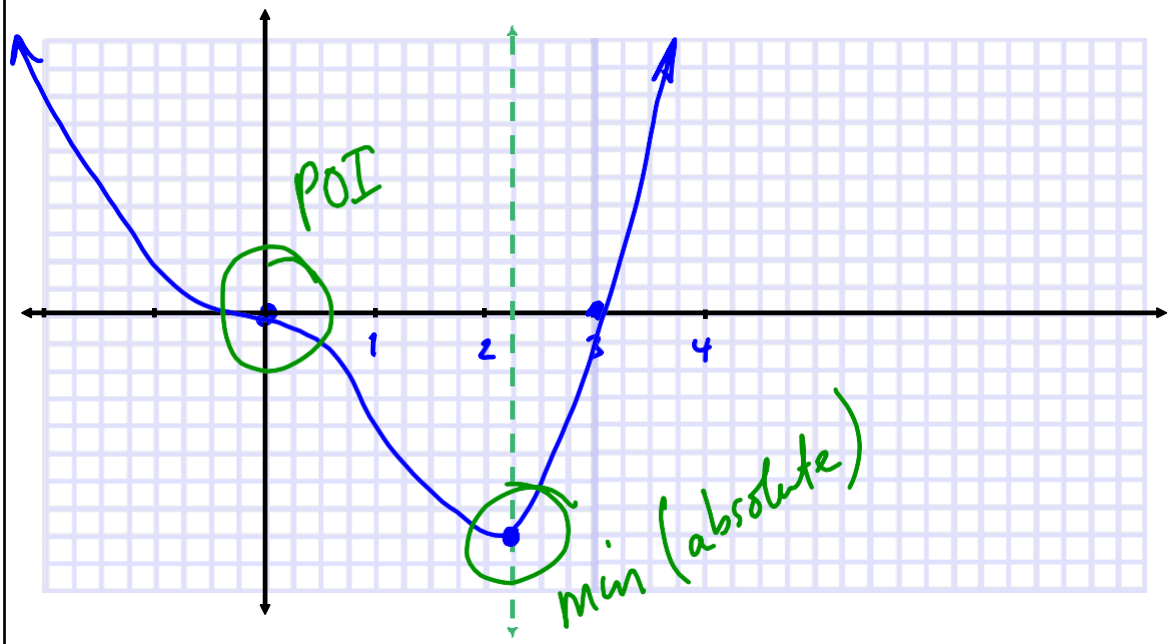
POI at $x=0$

$P(0,0)$

local min at $(\frac{9}{4}, -\frac{2187}{256})$

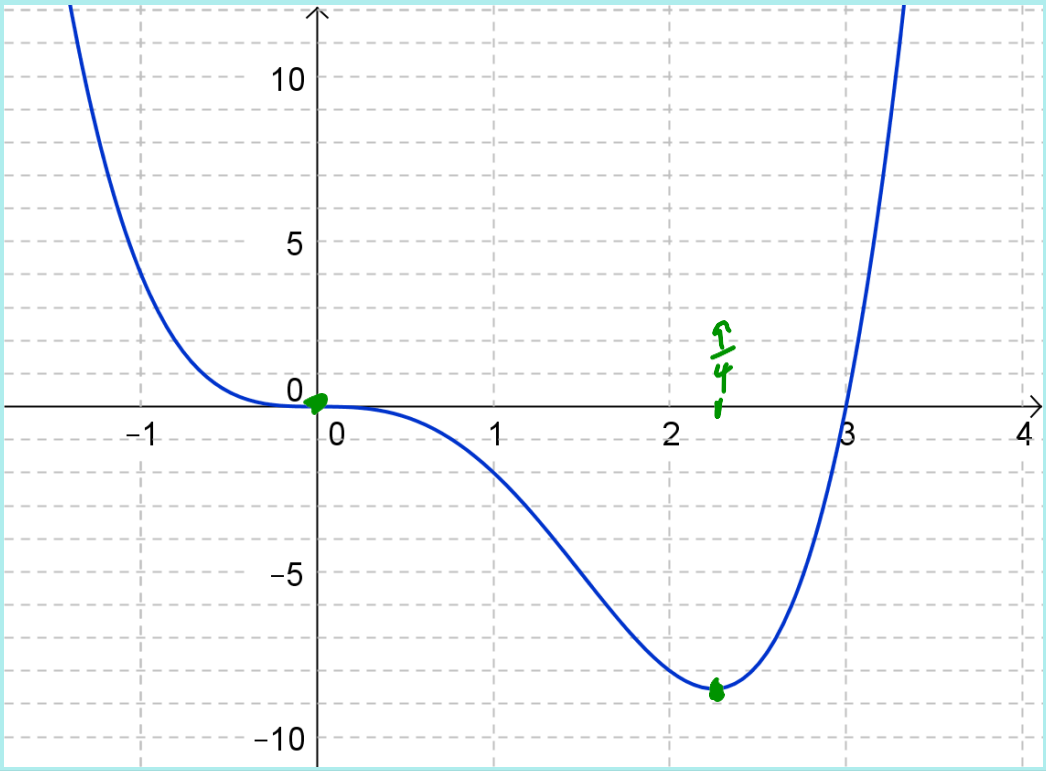
Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.

d) Sketch $f(x)$.



Ex.1 Consider the function defined by $f(x) = x^4 - 3x^3$.

d) Graph *f(x)*.



Ex.2 Find the critical point(s) of the function

$f(x) = \sqrt[5]{(x+3)^2}$

$= \left[(x+3)^2\right]^{\frac{1}{5}}$

$= (x+3)^{\frac{2}{5}}$

$f'(x) = \frac{2}{5}(x+3)^{-\frac{3}{5}}(1)$

for critical values, set $f'(x) = 0$

$0 = \frac{2}{5\sqrt[5]{(x+3)^3}}$

cross multiply

$0 = 2$???

$\therefore$ no solution

also: $5\sqrt[5]{(x+3)^3} \neq 0$

$x+3 \neq 0$

$x \neq -3$

in $f'(x)$

but $f(-3) = \sqrt[5]{(-3+3)^2}$

$= 0$

$f(-3)$ exists

$f'(-3)$ ~~DNE~~ undefined

$\downarrow$

$x = -3$ is a critical value

test $x \to -3^-$, $x \to -3^+$

$f(-2) = 1$

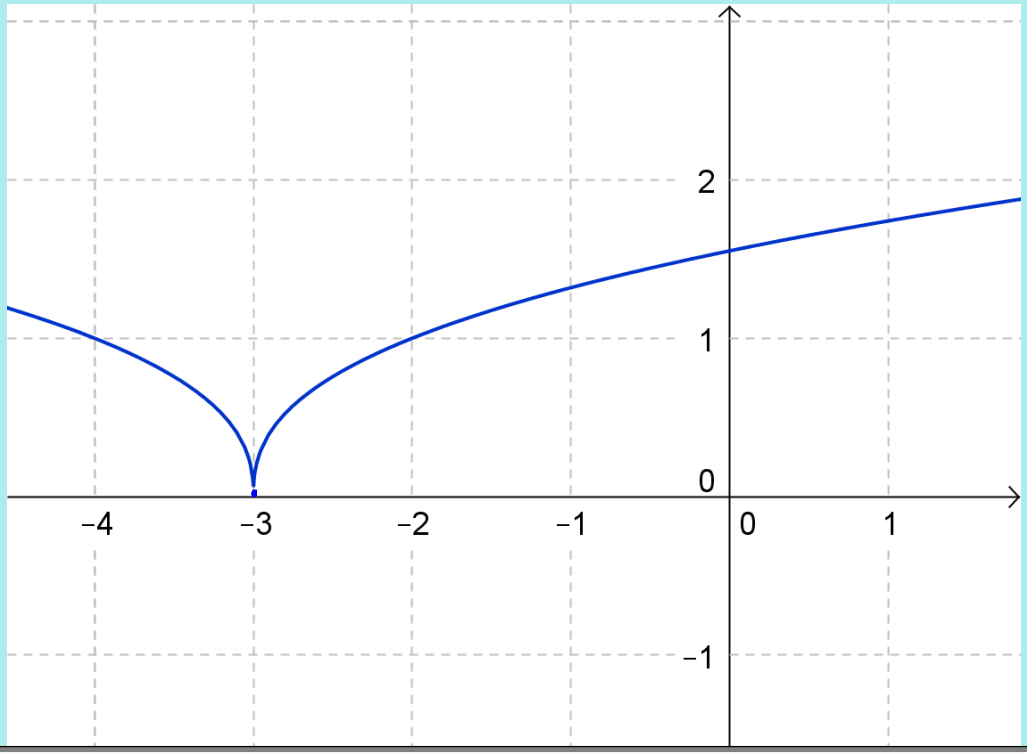
$f(-4) = 1$

$(-3,0)$ is a critical point (a cusp)

$\therefore (-3,0)$ is a minimum.

Ex.2 Find the critical point(s) of the function

$$f(x) = \sqrt[5]{(x+3)^2}$$



Assigned Work:

p.178 # 3, 4, 5bc, 7cd