3.3 – Concavity and the Second Derivative Test

Goal: To perform a second derivative test to determine points of inflection and whether functions are concave up or concave down.

We've seen that a function has a ***critical point*** at $(a, f(a))$ when _________________________.

Right now, to classify critical points, we need to check the behaviour of the original function around the critical point. Instead, we can use a **second derivative test** to easily classify critical points.

A function $f(x)$ is said to be **concave *up*** if:

A function $f(x)$ is said to be **concave *down*** if:

If $f''(a) = 0$,

The Second Derivative Test	
If $f'(a) = 0$ and $f''(a) > 0$ • $f(x)$ is **concave *up*** • $(a, f(a))$ is a **local *minimum***	
If $f'(a) = 0$ and $f''(a) < 0$ • $f(x)$ is **concave *down*** • $(a, f(a))$ is a **local *maximum***	
If $f''(x)$ changes sign at *a,* • $(a, f(a))$ is an ***inflection point*** If $f'(a) = 0$ as well, then • $(a, f(a))$ is called a ***saddle point***	

Example 1. (p.174 #7c) Find the critical points of the function $f(x) = x^4 - 6x^2 + 10$. Then, classify them using the second derivative test.

Example 2. (Similar to p.174 #5) Find the inflection points and the intervals of concavity for the function $f(x) = x^4 - 6x^2 - 5$.

Second Derivative Test:

Interval					
T.V.					
$f''(x)$					
$f(x)$					

Inflection Point(s):

$f(x)$is **concave *up*** (C.U.) when:

$f(x)$is **concave *down*** (C.U.) when:

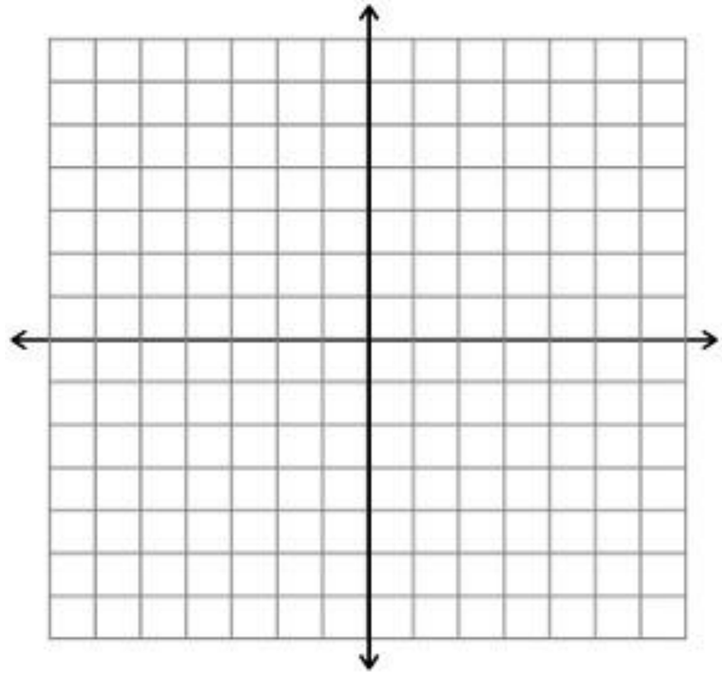
Example 3. (p. 174 #6f) Sketch a graph of a function that satisfies the set of conditions:

$f''(x) < 0$ when $-2 < x < 1$

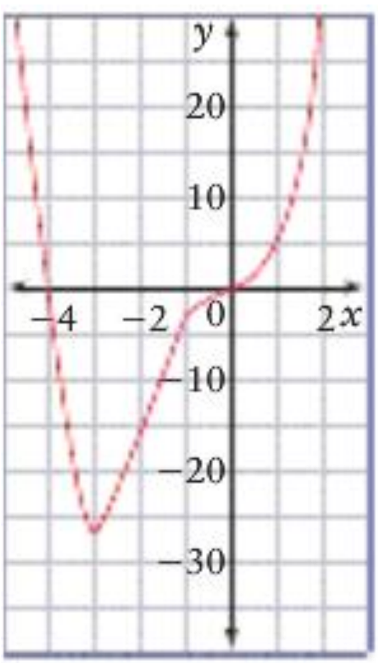
$f''(x) > 0$ when $x < -2$ and $x > 1$

$f(-2) = -3$

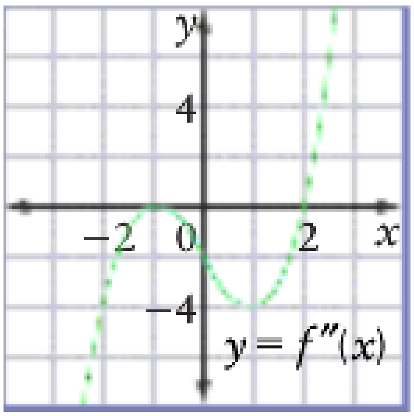
$f(0) = 0.$

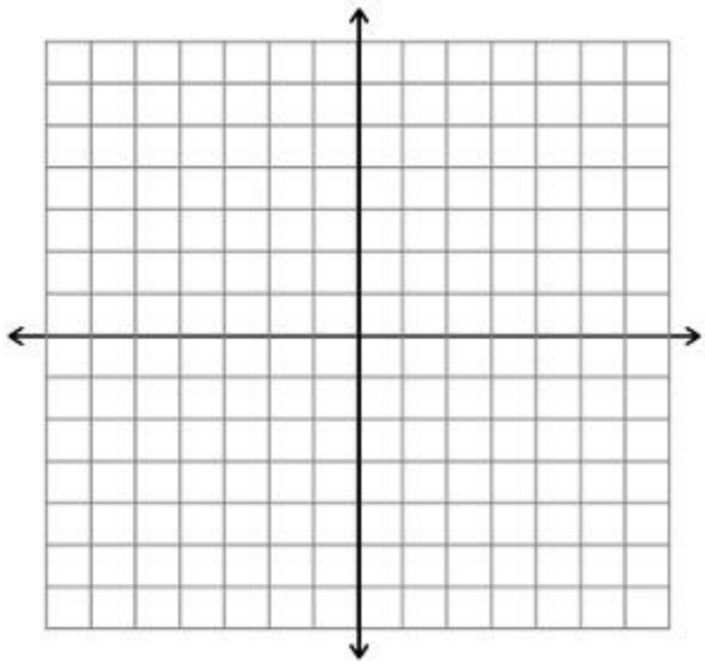


Example 4. (p.174 #1b) For the graph shown to the right, identify the intervals over which the graph is concave up and the intervals over which it is concave down.



Example 5. (p.174 #2d,3d) Given the graph of $f''(x)$, state the intervals of concavity for the function $f(x)$. Also indicate where any inflection points occur for $f(x)$. Then, sketch a possible graph of $y = f(x)$.





IP - P. 173 #1-3, 5c, 6, 7