MCV4U – 3.3 (Part A) Date: COMPLETED NOTE.

3.3 – Concavity and the Second Derivative Test

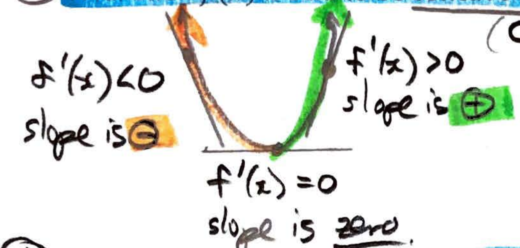
Goal: To perform a second derivative test to determine ~~points of inflection~~ inflection points (IPs) and whether functions are concave up or concave down.

We've seen that a function has a ***critical point*** at $(a, f(a))$ when $f'(a) = 0$ or $f'(a) = \phi$ or D.N.E.

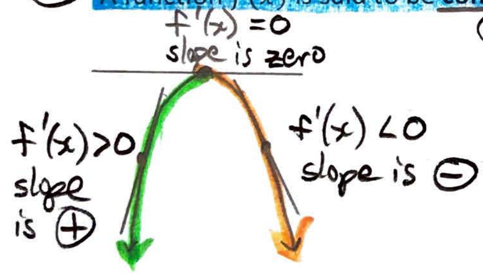
i.e. first derivative test

Right now, to classify critical points, we need to check the behaviour of the original function around the critical point. Instead, we can use a **second derivative test** to easily classify critical points.

- A function $f(x)$ is said to be **concave *up*** if: (C.U.) the rate of change of the slope is positive. $\boxed{f''(x) > 0}$ i.e. the slope gets more positive



- A function $f(x)$ is said to be **concave *down*** if: (C.D.) the rate of change of the slope is negative $\boxed{f''(x) < 0}$ i.e. the slope gets more negative.



- If $f''(a) = 0$, If $f''(a) = 0$, there might be an Inflection Point (I.P.)

At an IP, a function changes concavity from CU to CD or from CD to CU.

To check if there's an IP, check if $f''(x)$ changes sign at $x = a$. (i.e. check if the concavity changes)

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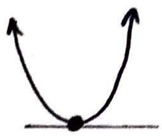
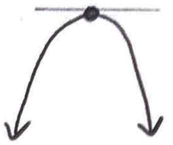
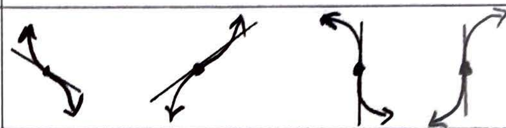
The Second Derivative Test	
If $f'(a) = 0$ and $f''(a) > 0$ • $f(x)$ is **concave up** • $(a, f(a))$ is a **local *minimum*** (type of C.P.)	 C.U.
If $f'(a) = 0$ and $f''(a) < 0$ • $f(x)$ is **concave down** • $(a, f(a))$ is a **local *maximum*** (type of C.P.)	 C.D.
If $f''(x)$ changes sign at a, • $(a, f(a))$ is an ***inflection point***	
If $f'(a) = 0$ as well, then • $(a, f(a))$ is called a ***saddle point*** (type of C.P.) (also type of I.P.)	 saddle points

inflection points.

Example 1. (p.174 #7c) Find the critical points of the function $f(x) = x^4 - 6x^2 + 10$. Then, classify them using the second derivative test.

① Find CPs ($f'(x) = 0$)

$f(x) = x^4 - 6x^2 + 10$

$f'(x) = 4x^3 - 12x$

$0 = 4x^3 - 12x$

$0 = 4x(x^2 - 3)$

$4x = 0$ → $x = 0$, $f(0) = 10$

$x^2 - 3 = 0$ → $x^2 = 3$ → $x = \pm\sqrt{3}$

$f(\sqrt{3}) = 9 - 6(3) + 10 = 1$

$f(-\sqrt{3}) = 9 - 6(3) + 10 = 1$

$\boxed{\text{CPs: } (0, 10)\ (\sqrt{3}, 1)\ (-\sqrt{3}, 1)}$

② Second derivative test
↳ concavity ($f''(x)$) changes

$f''(x) = 12x^2 - 12$

	$x = -\sqrt{3}$	$x = 0$	$x = \sqrt{3}$
$f''(x)$	24 (+)	−12 (−)	24 (+)
$f(x)$ info	CU ∴ CP is min $(-\sqrt{3}, 1)$	CD ∴ CP is max $(0, 10)$	CU ∴ CP is min $(\sqrt{3}, 1)$

$\boxed{\text{Homework: p.173 \#1, 2, 7}}$