Derivatives of Exponential Functions

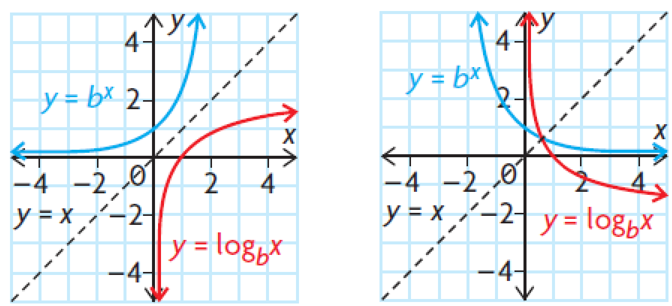
Many mathematical relations in the world are nonlinear. We have already looked at many polynomial and rational functions and their rates of change. Another type of nonlinear model is the exponential function. Exponential functions are often used to model rapid change. Compound interest, population growth, radioactive decay are just a few examples.

Recall:

Properties of Exponents

- $b^m b^n = b^{m+n}$
- $\dfrac{b^m}{b^n} = b^{m-n}, b^n \neq 0$
- $(b^m)^n = b^{mn}$
- $b^{\log_b m} = m$
- $\log_b b^m = m$

Graphs of $y = \log_b x$ and $y = b^x$



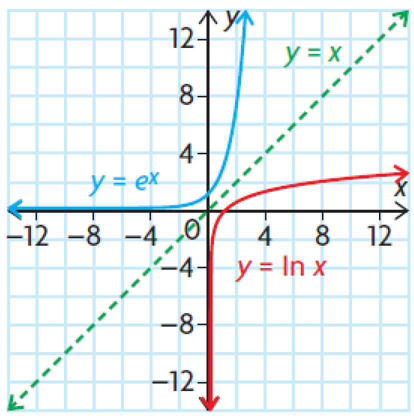
for $b > 1$ for $0 < b < 1$

Properties of the Exponential Function, $y = b^x$

- The base b is positive and $b \neq 1$.
- The y-intercept is 1.
- The x-axis is a horizontal asymptote.
- The domain is the set of real numbers, **R**.
- The range is the set of positive real numbers.
- The exponential function is always increasing if $b > 1$.
- The exponential function is always decreasing if $0 < b < 1$.
- The inverse of $y = b^x$ is $x = b^y$.
- The inverse is called the logarithmic function and is written as $\log_b x = y$.

Today we are going to look at a special exponential function $y = e^x$ and it's derivative. The number "e" is called the natural number, or Euler's number. It has an approximate value of 2.718

Since $y = e^x$ is an exponential function, it has the same properties as other exponential functions that we have studied. Like other exponential functions, $y = e^x$ also has an inverse. It's inverse is $y = \log_e x$ and just like other exponential functions $y = e^x$ and $y = \log_e x$ are reflections in the line y = x. The function $y = \log_e x$ can also be written as $y = \ln x$ and is called the natural logarithm function.



All the properties of exponential functions and logarithmic functions that we have learned previously still apply to the above mentioned functions.

$y = e^x$	$y = \ln x$
• The domain is $\{x \in \mathbf{R}\}$.	• The domain is $\{x \in \mathbf{R} \mid x > 0\}$.
• The range is $\{y \in \mathbf{R} \mid y > 0\}$.	• The range is $\{y \in \mathbf{R}\}$.
• The function passes through (0, 1).	• The function passes through (1, 0).
• $e^{\ln x} = x,\ x > 0$.	• $\ln e^x = x,\ x \in \mathbf{R}$.
• The line $y = 0$ is the horizontal asymptote.	• The line $x = 0$ is the vertical asymptote.

The derivative of $f(x)=e^x$ is $f'(x)=e^x$. This means that the slope of the tangent at any given point is the value of the function at this point.

Derivative of a Composite Function Involving e^x

In general, if $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)}g'(x)$ by the chain rule.

Example # 1: Determine the derivative of $f(x)=e^{8x}$.

Example #2: Determine the derivative of;

a) $f(x)=e^{3x^2-4x+1}$

b) $f(x)=x^3e^{2x}$

Example # 3: Given $f(x) = -5e^{x^2}$, determine $f'(-1)$

Example #4:

Determine the equation of the line tangent to $y = \frac{e^x}{x^2}$, where $x = 2$.

Homework: p.232 # 2ef , 3cdef, 4b , 6 – 8 , 10ab , 11c , 12 , 13ab