Name: **Date:**

7 – The Derivative of Natural Logarithm and General Logarithmic Functions

Lesson Goals:

- Be able to apply the differentiation rules to find the derivative of logarithmic functions
- Be able to determine the derivative of a logarithmic function with any constant base

1) Derivative of $y = \ln x$

- Recall that the inverse of $y = e^x$ is $y = \ln x$.
- $y = \ln x$ can be rewritten as $y = log_e x$ which can be rewritten at $e^y = x$.
- Let's find $\frac{dy}{dx}$:

- The derivative of the natural logarithmic function $y = \ln x$ is $\frac{dy}{dx} = \frac{1}{x}$, $x > 0$.
- If $f(x) = \ln(g(x))$, then $f'(x) = \frac{1}{g(x)} g'(x)$ or $f'(x) = \frac{g\prime(x)}{g(x)}$, by the chain rule.

Example 1: Differentiate.

$$y = \ln(3x^4 - 5x)$$

Example 2: Determine the derivative for each function. Express your answers in factored form.

a) $f(x) = 5x^2 \ln\sqrt{x}$

b) $g(x) = \frac{\ln(x-2)}{3x}$

Example 3: Determine all the points on the graph of $h(x) = (x\ln x)^2$ that have a horizontal tangent line.

2) Derivative of $y = \log_a x, a > 0$

- Let's find the derivative for a general logarithmic function:

$$y = log_a x, a > 0$$

- For $y = log_a x, a > 0, a \neq 1, f'(x) = \frac{1}{x\ln a}$
- For $y = log_a g(x), a > 0, a \neq 1, f'(x) = \frac{g'(x)}{g(x)\ln a}$

Example 4: Differentiate.

$$y = log_5(3x^4 + x)$$

Example 5: Determine the derivative for each function. Express your answers in factored form.

a) $f(x) = \left(log_3(5x^2 - x)\right)^4$

b) $g(x) = log_3(5x^2 - x)^4$

Example 6: If $f(x) = log_3(4x - x^2) + 2x$. At what point is the tangent ot the curve $y = f(x)$ perpendicular to the line $x + 2y - 6 = 0$?

Homework: Page 575 #3-11 and Page 578 #1-9 (pick and choose)