6.1,6.2 Intro to Vectors, Vector Addition and Subtraction Completed Note

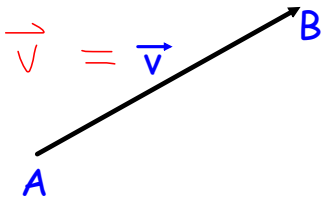
6.1 Introduction to Vectors

Scalar	vs.	Vector
a quantity that has only magnitude		a quantity that has both magnitude AND direction
mass		velocity
area		friction
speed		weight
age		torque
temperature		

Characteristics of Vectors:

1. A vector can be represented by a directed line segment:

"A" is the tail of the vector. (starting point)
"B" is the head of the vector. (ending point)
It is called $\overrightarrow{AB}$, or "vector AB".
The magnitiude is denoted by $|AB|$.

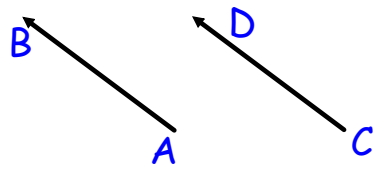


Vectors are also named using lower case letters. (u,v,w are common)
In this case $\vec{v} = \overrightarrow{AB}$

Eg. IF $\vec{v}$ represented the velocity of an airplane, the direction of the arrow would represent the direction of the plane and the length would represent its speed.

2. Equal Vectors

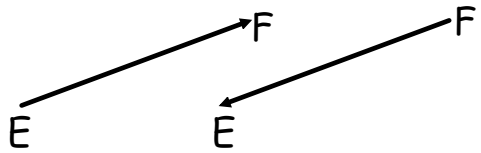
Two vectors are equal if they are parallel to each other and have the same direction AND if the magnitudes are equal.



$\overrightarrow{AB} = \overrightarrow{CD}$

3. Opposite Vectors

Two vectors are opposite if they have the same magnitude but point in opposite directions.



$\overrightarrow{FE}$ and $\overrightarrow{EF}$ are opposites.

$\overrightarrow{FE} = -\overrightarrow{EF}$

$|\overrightarrow{FE}| = |\overrightarrow{EF}|$

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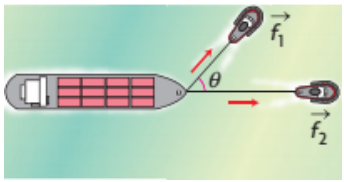
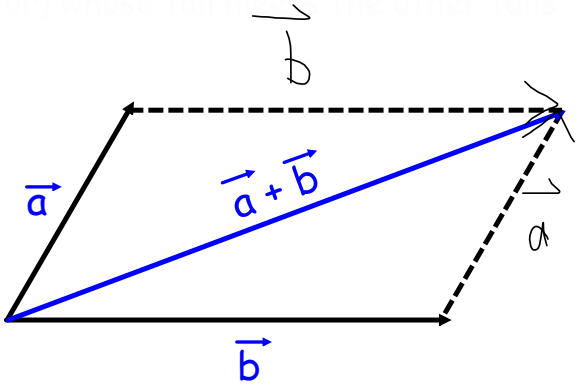
6.2 Vector Addition



The Parallelogram Law for Adding Two Vectors

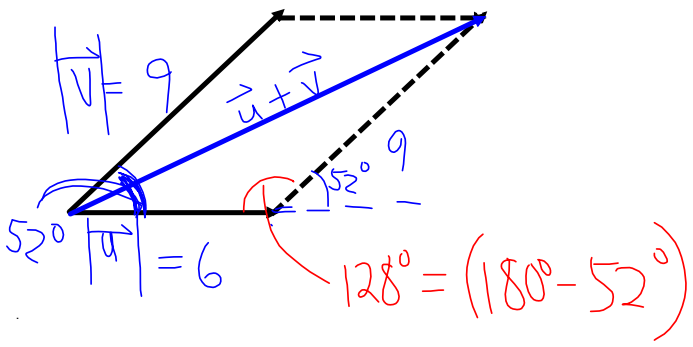
- place the vectors tail to tail
- complete the parallelogram formed by the two vectors
- the sum or resultant is the diagonal of the parallelogram (also a vector) whose tail meets the other tails



$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

Ex. 1 Given vectors $\vec{u}$ and $\vec{v}$ such that the angle between them is 52°, $|\vec{u}| = 6$ and $|\vec{v}| = 9$, determine $|\vec{u} + \vec{v}|$. Include a diagram.



$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos 128°$

$|\vec{u}+\vec{v}|^2 = 6^2 + 9^2 - 2(6)(9)\cos 128°$

$|\vec{u}+\vec{v}| \doteq 13.5$ units

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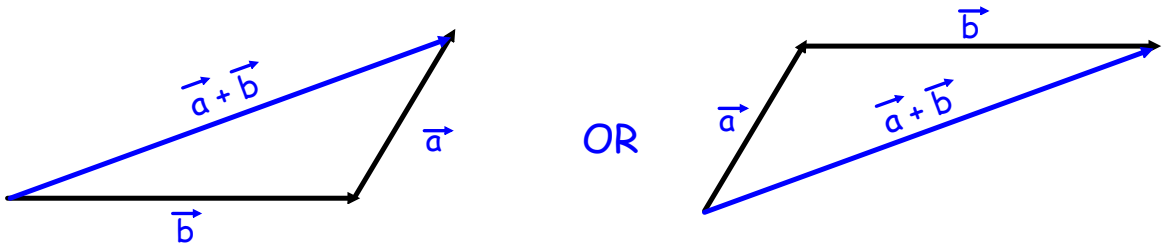
The Triangle Law of Addition

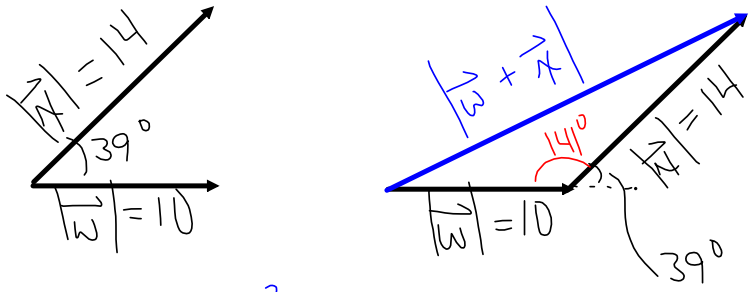
- translate one of the vectors so that the vectors are head to tail
- the sum or resultant is the vector that goes from the tail of the first to the head of the second vector



Note:

determining the vector $\vec{a} - \vec{b}$ is equivalent to finding $\vec{a} + (-\vec{b})$

Ex. 2 Given vectors, $\vec{w}$ and $\vec{x}$ such that the angle between them is 39°, $|\vec{w}| = 10$ and $|\vec{x}| = 14$, determine $|\vec{w} + \vec{x}|$. Include a diagram.



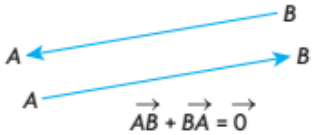
$$|\vec{w} + \vec{x}|^2 = |\vec{w}|^2 + |\vec{x}|^2 - 2|\vec{w}||\vec{x}|\cos 141^\circ$$

$$|\vec{w} + \vec{x}|^2 = 10^2 + 14^2 - 2(10)(14)\cos 141^\circ$$

$$|\vec{w} + \vec{x}| \doteq 22.7$$

The Zero Vector

When two opposite vectors are added, the resultant is the zero vector.
The zero vector has a magnitude of 0, ie $|\vec{0}| = 0$ and no defined direction.



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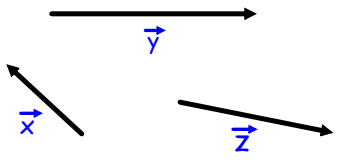
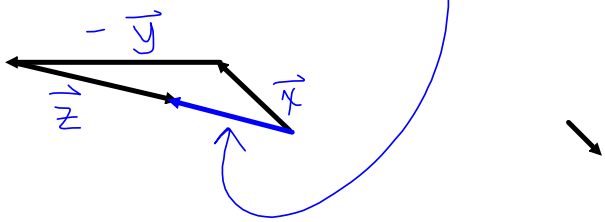
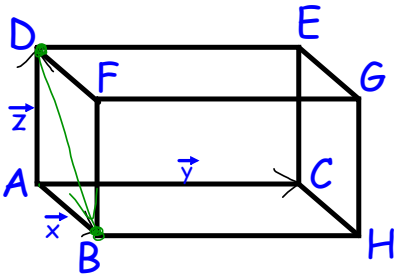
Ex3 Given the vectors shown, construct one vector equivalent to $\vec{x} - \vec{y} + \vec{z}$





Ex4 Express each of the following in terms of $\vec{x}$, $\vec{y}$ and $\vec{z}$, where $\vec{x} = \overrightarrow{AB}$, $\vec{y} = \overrightarrow{AC}$ and $\vec{z} = \overrightarrow{AD}$.



a) $\overrightarrow{BH} = \vec{y}$

b) $\overrightarrow{EC} = -\vec{z}$

c) $\overrightarrow{DB} = -\vec{z} + \vec{x}$

d) $\overrightarrow{HA} = -\vec{y} - \vec{x}$

e) $\overrightarrow{BE} = \vec{y} - \vec{x} + \vec{z}$

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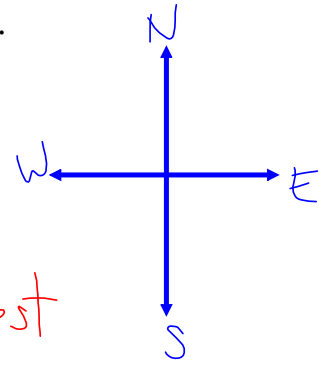
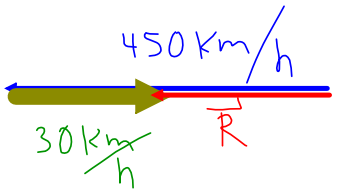
Ex5 A plane is travelling due West at 450 km/h. directly
The velocity of the plane is affected by the direction and speed of the wind.
Determine the resultant ground velocity for each case.



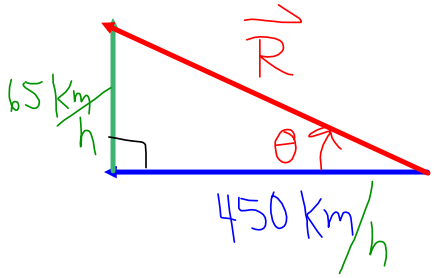
a) The wind is from the west at 30 km/h.



$|\vec{R}| = 450 - 30$
$= 420 \frac{km}{h}$ due west

$\vec{R}$ = resultant vector

b) The wind is from the south at 65 km/h.



$|\vec{R}|^2 = 450^2 + 65^2$
$|\vec{R}| \doteq 454.7 \text{ km/h}$

$\tan\theta = \frac{65}{450}$

$\theta = \tan^{-1}\left(\frac{65}{450}\right)$

$\theta \doteq 8.2^\circ$

$\therefore \vec{R}$ is $454.7 \frac{km}{h}$ W 8.2° N

6.1/6.2 Assignment:
p. 281 #9
p. 290 #3ab, 5, 9, 11, 12