Properties of Vectors

Apr. 24/2014

Vector Addition:

Commutative Property $\vec{a}+\vec{b}=\vec{b}+\vec{a}$

Associative Property $\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)$

Distributive Property $k\left(\vec{a}+\vec{b}\right)=k\vec{a}+k\vec{b}$, where $k\in\mathbb{R}$

Adding Zero $\vec{a}+\vec{0}=\vec{a}$

Scalar Multiplication:

Associative Law $m\left(n\vec{a}\right)=mn\vec{a}$

Distributive Law $\left(m+n\right)\vec{a}=m\vec{a}+n\vec{a}$

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Ex.1 Given that $\vec{x}=\vec{i}+3\vec{j}-2\vec{k}$

$\vec{y}=\vec{j}+5\vec{k}$

$\vec{z}=4\vec{i}-\vec{j}-7\vec{k}$

determine a simplified expression for $\vec{x}-\vec{y}+3\vec{z}$

in terms of $\vec{i}$, $\vec{j}$, and $\vec{k}$

$\vec{x}-\vec{y}+3\vec{z}$

$=(\vec{i}+3\vec{j}-2\vec{k})-(\vec{j}+5\vec{k})+3(4\vec{i}-\vec{j}-7\vec{k})$

$=\vec{i}+3\vec{j}-2\vec{k}-\vec{j}-5\vec{k}+12\vec{i}-3\vec{j}-21\vec{k}$

$=13\vec{i}-\vec{j}-28\vec{k}$

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Collinear Vectors

Two vectors are said to be collinear if and only if:

$$\vec{a} = k\vec{b}, \text{ where } k \in \mathbb{R}$$

The scalar k is a scale factor, which can stretch, compress, or reflect the original vector.

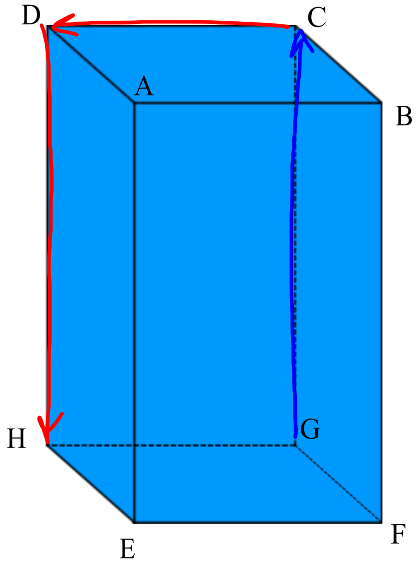
Unit Vector

A unit vector is collinear to a given vector, having a magnitude of one.

Given vector $\vec{v}$, the corresponding unit vector is $\vec{u} = \frac{\vec{v}}{|\vec{v}|}$

Apr 23-3:00 PM

Ex.2 Write an equivalent vector and show it on the diagram.



(a) $\overrightarrow{EG} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{DC}$

$= \overrightarrow{EC}$

(b) $\overrightarrow{BD} + \overrightarrow{DA} + \overrightarrow{AB}$

$= \overrightarrow{0}$

(c) $\overrightarrow{GC} + \overrightarrow{FE} - \overrightarrow{FB}$

$= \overrightarrow{GC} + \overrightarrow{CD} + \overrightarrow{BF}$

$= \overrightarrow{GC} + \overrightarrow{CD} + \overrightarrow{DH}$

$= \overrightarrow{GH}$

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Assigned Work:

p.299 # 1, 5, 7, 12, 14, 15, 19, 21*

p.306 # 5, 7, 8, 10, 11

p.299
7. $\vec{a} = \frac{2}{3}\vec{b}$ $\quad \vec{a} = \frac{1}{2}\vec{c}$

$\vec{b} = \frac{3}{2}\vec{a}$ $\quad \vec{c} = 2\vec{a}$

(a) $m\vec{c} + n\vec{b} = \vec{0}$

$m(2\vec{a}) + n\left(\frac{3}{2}\vec{a}\right) = \vec{0}$

$2m\vec{a} + \frac{3}{2}n\vec{a} = \vec{0}$

$\left(2m + \frac{3}{2}n\right)\vec{a} = \vec{0}$

$\left(2m + \frac{3}{2}n\right)\vec{a} = 0\vec{a}$

$\Rightarrow 2m + \frac{3}{2}n = 0 \quad \times 2$

$4m + 3n = 0$

$4m = -3n$

$\boxed{m = \frac{-3}{4}n}$

-6, 8
-3, 4
0, 0
3, -4
6, -8
9, -12

m & n must be integers
$\therefore$ n must be a multiple of 4

$\Rightarrow$ m is always a multiple of 3.

There are an infinite # of possibilities

(b) $d\vec{a} + e\vec{b} + f\vec{c} = \vec{0}$

$d\vec{a} + e\left(\frac{3}{2}\vec{a}\right) + f(2\vec{a}) = \vec{0}$

$\left(d + \frac{3}{2}e + 2f\right)\vec{a} = 0\vec{a}$

$d + \frac{3}{2}e + 2f = 0 \quad \times 2$

$2d + 3e + 4f = 0$

3 2 -3

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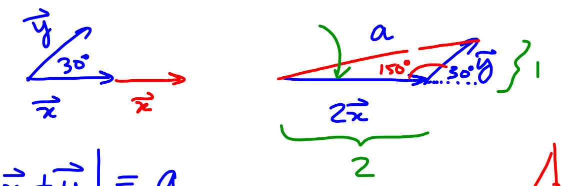
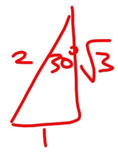
p.299 #15.



$|2\vec{x} + \vec{y}| = a$



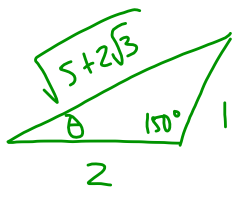
$a^2 = 2^2 + 1^2 - 2(2)(1)\cos 150°$

$a^2 = 5 - 4\left(-\frac{\sqrt{3}}{2}\right)$

$a^2 = 5 + 2\sqrt{3}$

$a = \sqrt{5 + 2\sqrt{3}}, \; a > 0$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$



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p. 306 #10

$\vec{x} = \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z}$ ①

$\vec{x} - \vec{y} = \vec{a}$ ②

$\vec{y} - \vec{z} = \vec{b}$ ③

show $\vec{a} = -\frac{1}{3}\vec{b}$

sub ① into ②: $\left(\frac{2}{3}\vec{y} + \frac{1}{3}\vec{z}\right) - \vec{y} = \vec{a}$

$-\frac{1}{3}\vec{y} + \frac{1}{3}\vec{z} = \vec{a}$ ×3

$-\vec{y} + \vec{z} = 3\vec{a}$

③ $\vec{y} - \vec{z} = \vec{b}$

$\vec{0} + \vec{0} = 3\vec{a} + \vec{b}$

$-\vec{b} = 3\vec{a}$

$-\frac{1}{3}\vec{b} = \vec{a}$

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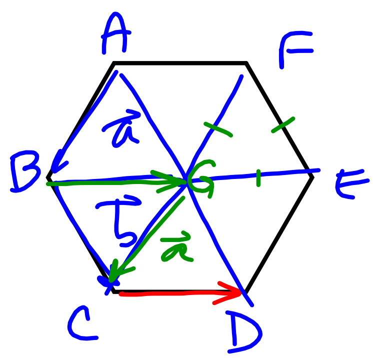
21.



$\vec{CD} = \vec{BG}$

$= \vec{b} - \vec{a}$

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