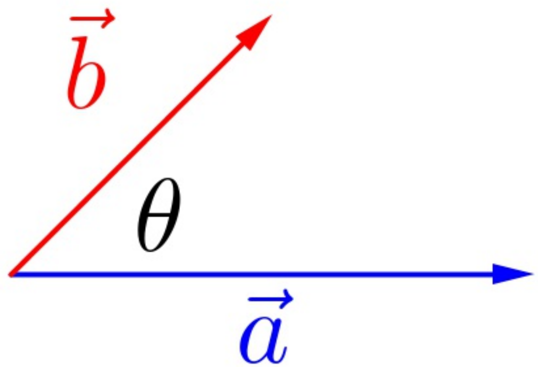
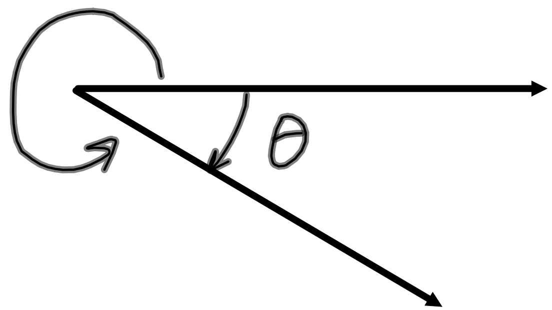
The Dot Product of Geometric Vectors

The dot product is one type of vector multiplication,
but the product itself (i.e., the result) is a scalar.



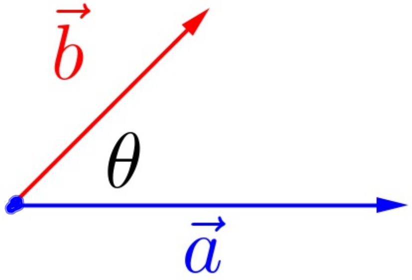


"a dot b"

$$\vec{a} \cdot \vec{b} = |\vec{a}| \left|\vec{b}\right| cos\theta \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ$$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$+ \quad +$

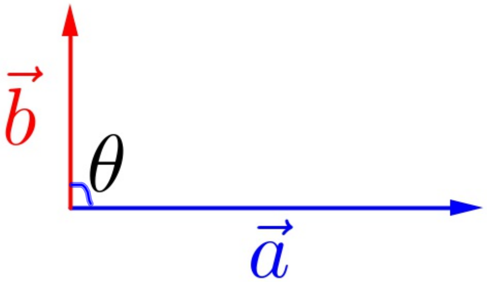


$$0^\circ \leq \theta < 90^\circ$$

$$cos\theta > 0$$

$$\therefore \vec{a}\cdot\vec{b} > 0$$

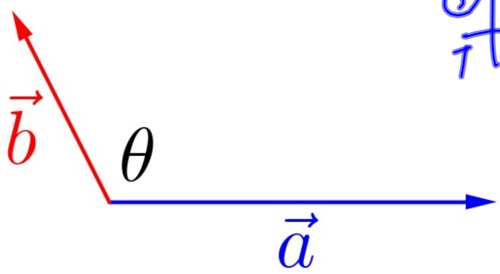
S A
T C



$$\theta = 90^\circ \quad cos90^\circ = 0$$

$$\therefore \vec{a}\cdot\vec{b} = 0$$

Can be used to test for right angles.



S A
T C

$$90^\circ < \theta \leq 180^\circ$$

$$cos\theta < 0$$

$$\therefore \vec{a}\cdot\vec{b} < 0$$

Properties of the Dot Product:

(1) Commutative: $$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

(2) Distributive:

vector

$$\vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$$

vector

(3) Magnitudes: $$\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$$

$$\vec{a}\cdot\vec{a}$$
$$=|\vec{a}||\vec{a}|\cancel{\cos 0}\ 1$$
$$=|\vec{a}|^2$$

(4) Associative with scalar:

$$\left(k\vec{a}\right)\cdot\vec{b}=\vec{a}\cdot\left(k\vec{b}\right)=k\left(\vec{a}\cdot\vec{b}\right)$$

Ex.2 Show that

$$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=|\vec{a}|^2-\left|\vec{b}\right|^2$$

$$LS=(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$$

$$=\vec{a}\cdot(\vec{a}-\vec{b})+\vec{b}\cdot(\vec{a}-\vec{b})$$

$$=\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}$$

$$=|\vec{a}|^2\underbrace{-\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}}_{0}-|\vec{b}|^2$$

$$=|\vec{a}|^2-|\vec{b}|^2 \qquad LS=RS \checkmark$$

Ex.1 Find the angle between vectors **u** and **v** given:

(1) $|\vec{u}| = 3\,|\vec{v}|$ $\qquad |\vec{u}| \neq 0 \quad |\vec{v}| \neq 0$

(2) $3\vec{u} + \vec{v}$ and $\vec{u} - 8\vec{v}$ are perpendicular.

dot product is 0.

FOIL

$$(3\vec{u}+\vec{v})\cdot(\vec{u}-8\vec{v}) = 0$$

$$3\vec{u}\cdot\vec{u} - 24\,\vec{u}\cdot\vec{v} + \underbrace{\vec{v}\cdot\vec{u}}_{\vec{u}\cdot\vec{v}} - 8\,\vec{v}\cdot\vec{v} = 0$$

$$3|\vec{u}|^2 - 23\,\vec{u}\cdot\vec{v} - 8|\vec{v}|^2 = 0$$

$$3|\vec{u}|^2 - 23|\vec{u}||\vec{v}|\cos\theta - 8|\vec{v}|^2 = 0$$

$$3\,(3|\vec{v}|)^2 - 23\,(3|\vec{v}|)|\vec{v}|\cos\theta - 8|\vec{v}|^2 = 0 \qquad [\div|\vec{v}|^2]$$

$|\vec{v}| \neq 0$

$$3(9) - 23(3)\cos\theta - 8 = 0$$

$$19 - 69\cos\theta = 0$$

$$19 = 69\cos\theta$$

$$\cos\theta = \frac{19}{69}$$

$$\theta \doteq 74.0^\circ$$

$\therefore$ the angle between $\vec{u}$ and $\vec{v}$ is 74.0°.

Assigned Work

p.377 #2, 5, 6abe, 7acd, 9, 11, 12