L1 – Equations of Lines in Two-Space Unit 6
MCV4U
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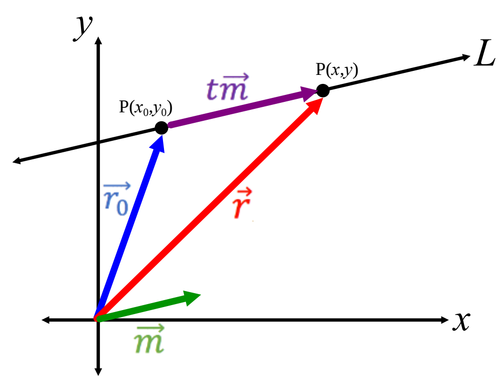
In two-space, a line can be defined many different types of equations:

i)	Slope y-intercept form	→	$y = mx + b$
ii)	Standard form (scalar equation)	→	$Ax + By + C = 0$
iii)	Vector Equation	→	$\vec{r} = \vec{r_0} + t\vec{m}$ or $[x, y] = [x_0, y_0] + t[m_1, m_2]$
iv)	Parametric Equation	→	$\begin{cases} x = x_0 + tm_1 \\ y = y_0 + tm_2 \end{cases}$

Part 1: Vector Equation of a Line in Two-Space

In slope y-intercept form and standard form, we have equations that define all points (x, y) on the line. A vector equation of a line is an equation that describes resultant vectors that start at the origin and end at a point on the line. In order to create these resultant vectors, we need a position vector that gives us a point on the line, $\vec{r_0} = [x_0, y_0]$, and a direction vector parallel to the line, $\vec{m} = [m_1, m_2]$. By adding the position and direction vectors together, we get a resultant vector, $\vec{r}$, that has its tip on a point on the line. By multiplying the direction vector by t (to get $t\vec{m}$), the resultant vector, $\vec{r}$, can have its tip at any point on the line.



Summary of Vector Equation of a Line:

$\vec{r} = \vec{r_0} + t\vec{m}$ or $[x, y] = [x_0, y_0] + t[m_1, m_2]$

where

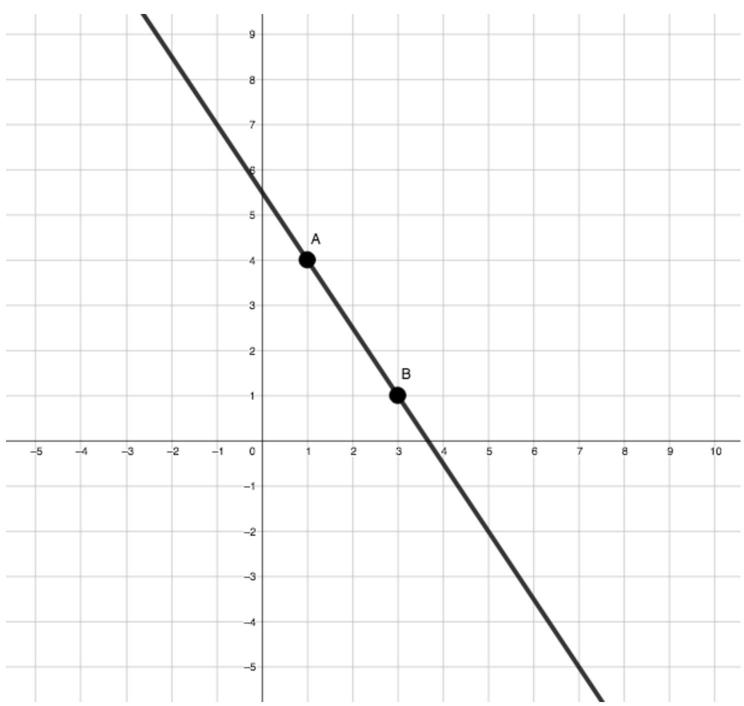
- $t \in \mathbb{R}$
- $\vec{r} = [x, y]$ is a position vector to any unknown point on the line
- $\vec{r_0} = [x_0, y_0]$ is a position vector to any known point on the line
- $\vec{m} = [m_1, m_2]$ is a direction vector parallel to the line

Example 1: For a line that goes through the points $A(1,4)$ and $B(3,1)$

a) Write a vector equation for the line

b) Determine three more position vectors to points on the line. Graph the line.

https://www.geogebra.org/graphing/j6ntqajv



c) Determine if the point (2,3) is on the line.

Part 2: Parametric Equation of a Line in Two-Space

Remember that the Vector Equation of a line can be written as:

i) $\vec{r} = \overrightarrow{r_0} + t\vec{m}$ or $[x, y] = [x_0, y_0] + t[m_1, m_2]$

The vector equation of the line can be separated in to two parts, one for each variable.

The parametric equations of a line in two-space are:

$$x = x_0 + tm_1$$
$$y = y_0 + tm_2$$

The parametric equation of the line from example 1 would be:

Example 2: Consider the line $\ell_1: \begin{cases} x = 3 + 2t \\ y = -5 + 4t \end{cases}$

a) Find the coordinates of two points on the line.

b) Write a vector equation of the line.

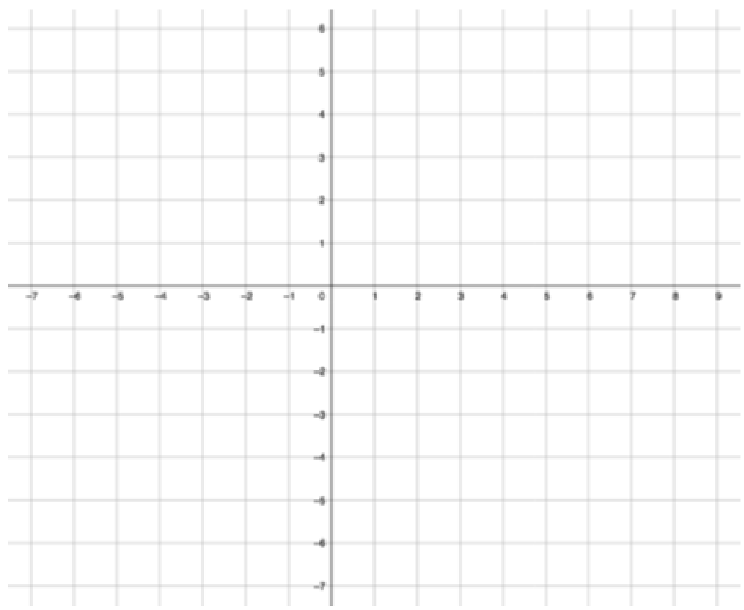
c) Write the scalar equation of the line.

To write the scalar equation, isolate t *in both parametric equations, then set them equal to each other and rearrange in to standard form.*

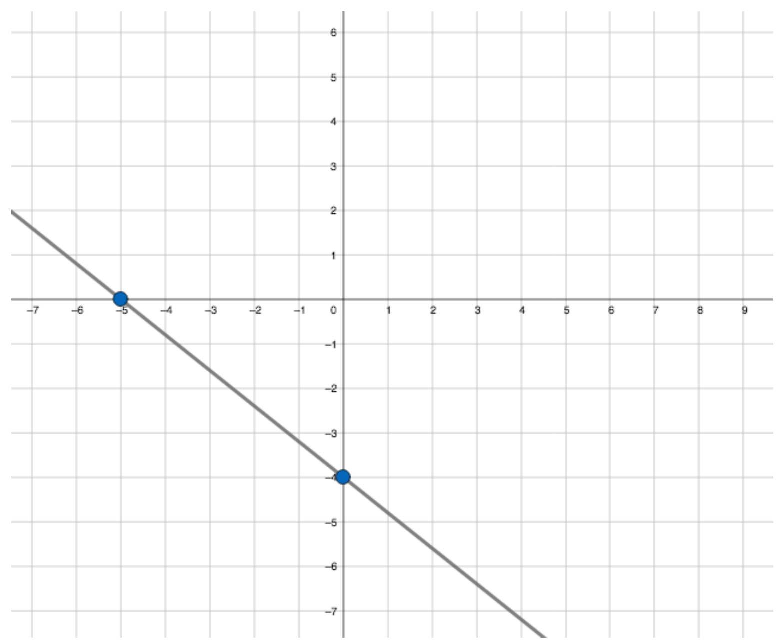
d) Determine if ℓ_1 is parallel to ℓ_2: $\begin{cases} x = 1 + 3t \\ y = 8 + 12t \end{cases}$

Example 3: Consider the line with scalar equation $4x + 5y + 20 = 0$

a) Graph the line



b) Determine a position vector that is perpendicular to the line.



c) How does the position vector from part b) compare to the scalar equation?

d) Write a vector equation of the line

Example 4: Find a scalar equation for the line $[x, y] = [0,3] + t[2, -1]$

Remember shortcut for finding a perpendicular vector is to swap coordinates and change 1 sign.