L2 – Equations of Lines in Three-Space Unit 6
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https://www.geogebra.org/3d/aku6q7xs

In two-space lines can be represented using: vector equations, parametric equations, scalar equations, or an equation in slope y-intercept form.

A line in three space can be defined by a vector, parametric, or symmetric equation but not a scalar equation. In three-space, a scalar equation defines a plane. A plane is a two-dimensional flat surface that extends infinitely in all directions.

As in two-space, a line in three-space needs a position vector to a known point on the line and a direction vector in order to define it.

Equations of a line in R^3:

i) Vector Equation → $\vec{r} = \vec{r_0} + t\vec{m}$ or $[x, y, z] = [x_0, y_0, z_0] + t[m_1, m_2, m_3]$

ii) Parametric Equation → $\begin{cases} x = x_0 + tm_1 \\ y = y_0 + tm_2 \\ z = z_0 + tm_3 \end{cases}$

where $\vec{r_0}$ is the position vector and $\vec{m}$ is the direction vector.

iii) Symmetric Equation → $\frac{x-x_0}{m_1} = \frac{y-y_0}{m_2} = \frac{z-z_0}{m_3}$

The symmetric equation is derived from the parametric equations and solving for the t parameter in each component.

Example 1: A line passes through points $A(2, -1, 5)$ and $B(3, 6, -4)$.

a) Write a vector equation of the line.

b) Write parametric equations for the line.

c) Determine if the point $C(0,-15,9)$ lies on the line.

Example 2: Find Vector, Parametric, and Symmetric equations of a line that passes through points $A(2,-1,3)$ and $B(5,1,1)$.

Notice that you cannot write a scalar equation of a line in 3-space. This is reserved for defining planes which we will do next lesson.