THE ERINDALE ACADEMY
1576 Dundas Street West, Mississauga ON L5C 1E5
www.teacademy.ca Tel: 905-232-1576
Email: info@teacademy.ca

Teacher: Ms. Ella

Unit 2: Derivatives

Lesson 2.2: Derivatives of Exponential function, Trigonometric functions, and Logarithmic functions.

Part I: Exponential function (From Textbook section 5.1 and 5.2)

Key Ideas

- For $f(x) = e^x$, $f'(x) = e^x$.
  In Leibniz notation, $\frac{d}{dx}(e^x) = e^x$.
- For $f(x) = e^{g(x)}$, $f'(x) = e^{g(x)} \times g'(x)$.
  In Leibniz notation, $\frac{d(e^{g(x)})}{dx} = \frac{d(e^{g(x)})}{d(g(x))} \frac{d(g(x))}{dx}$.
- The slope of the tangent at a point on the graph of $y = e^x$ equals the value of the function at this point.

Need to Know

- The rules for differentiating functions, such as the product, quotient, and chain rules, also apply to combinations involving exponential functions of the form $f(x) = e^{g(x)}$.
- e is called Euler's number or the natural number, where $e \doteq 2.718$.

With different base other than e:

- If $f(x) = b^x$, then $f'(x) = b^x \times \ln b$.
  In Leibniz notation, $\frac{d}{dx}(b^x) = b^x \times \ln b$.
- If $f(x) = b^{g(x)}$, then $f'(x) = b^{g(x)} \times \ln b \times g'(x)$.
  In Leibniz notation, $\frac{d}{dx}(b^{g(x)}) = \frac{d(b^{g(x)})}{d(g(x))} \frac{d(g(x))}{dx}$.

Example 1: Determine the derivative of each function.

a) $g(x) = e^{x^2 - x}$

b) $f(x) = x^2 e^x$

c) $f(x) = (2)^{\frac{x^2}{e^x}}$

d) $h(x) = (\frac{1}{5})^{(\sqrt{x^2+1})(5^{x^2})}$

Example 2: Determine the equation of the line tangent to y $= \frac{e^x}{x^2}$, where x = 3.

Teacher: Ms. Ella

Part II: Trigonometric functions (From Textbook Section 5.4 and 5.5)

IN SUMMARY

Key Idea

- The derivatives of sinusoidal functions are found as follows:
  - $\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$
  - If $y = \sin f(x)$, then $\frac{dy}{dx} = \cos f(x) \times f'(x)$.
  - If $y = \cos f(x)$, then $\frac{dy}{dx} = -\sin f(x) \times f'(x)$.
- The derivatives of functions involving the tangent function are found as follows:
  - $\frac{d(\tan x)}{dx} = \sec^2 x$
  - $\frac{d}{dx}(\tan f(x)) = \sec^2 f(x) \times f'(x)$

Example 3: Determine $\frac{dy}{dx}$ for each function.

a) $y = cos3x$

b) $y = xsinx$

c) $y = sin^2x^2$

d) $y = tanx^2$

Example 4: Differentiating a composite or combination of functions of $y = e^{sinx+tanx}\cos(1+x^3)$.

Teacher: Ms. Ella

Example 5: Determine the equation of the tangent to the graph of $y = xcos2x \ at \ x = \frac{\pi}{2}$.

Example 6: Determine the maximum and minimum values of the function $f(x) = cos^2 x$ on the interval $x \in [0, 2\pi]$.

Teacher: Ms. Ella

Part III: Logarithmic functions (Textbook pg576 - 578)

Derivatives of logarithmic functions:

$\frac{d}{dx}(lnx) = \frac{1}{x}$ and $\frac{d}{dx}[ln\,(f(x)] = \frac{f\prime(x)}{f(x)}$

$\frac{d}{dx}(log_a x) = \frac{1}{lna \times x}$ and $\frac{d}{dx}[log_a\,(f(x)] = \frac{f\prime(x)}{lna \times f(x)}$

Example 7: Take the differentiation of the following composite functi

a) $y = ln\,(sinx + cosx)$

b) $y = \frac{1+log_{10} tan\,(1-3x)}{x^2}$

c) $y = tan\,(log\sqrt{x - sin^2 x})$

d) $y = log\,[log_3(log_5 x)]$

Teacher: Ms. Ella

Suggested problem from Textbook:

Pg84. #26

Exponential differentiation:
Section 5.1: pg232 – 234. # 2, 3, 4, 5ab, 7, 8, 9, 10a, 11, 12, 13, 15, 16, 17
Section 5.2: pg240. #1,2, 3, 4, 5, 8

Trigonometry differentiation:
Section 5.4: pg256 – 257. #1, 2, 3, 5, 7, 8, 9, 10, 11, 14. (12 and 13 will be covered in Unit 3)
Section 5.5: pg260. #1, 2, 3, 4, 5, 6, 8, 10, 11

Review exercise: pg263 – 265. #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 23 (19, 20, 21 and 22 will be covered in Unit 3)

Chapter 5 test: pg266. #1, 2, 3, 5, 6, 7 (8 and 9 will be covered in Unit 3)

Cumulative Review of Calculus: pg267 – 270. #1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 24, 25, 26, 27, 28, 30, (17, 18, 19, 20, 21, 22, 23, 31 and 32 will be covered in Unit 3)