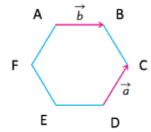
AS Assessment: Vector addition and subtraction

1. A regular hexagon, with sides of 3 cm, is shown below. Assume we place the x-y plane at vertex D, and D is the origin (0, 0). Determine:
   a) $\vec{a}+\vec{b}$ b) $\overrightarrow{FE}-\overrightarrow{CA}$



2. If $\vec{a}=(k,\frac{1}{2},k)$ is a unit vector, determine all possible values of k.

3. In triangle ABC, a median is drawn from vertex A to the midpoint of BC, which is labelled D. If $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{AC}=\vec{c}$, prove that $\overrightarrow{AD}=\frac{1}{2}\vec{a}+\frac{1}{2}\vec{c}$.

4. a) Draw a diagram on the appropriate coordinate system for the following vector: $\overrightarrow{OM}=(-1,3,-2)$.
   b) Determine the angles between vector $\overrightarrow{OM}$ and x-, y-, and z-axis.

5. Determine whether $\vec{r}=(16,11,-24)$ can be written as a linear combination of $\vec{p}=(-2,3,4)$ and $\vec{q}=(4,1,-6)$. Explain the significance of your result.

#1

A regular hexagon, with sides of 3 cm, is shown below. Assume we place the x-y plane at vertex D, and D is the origin (0, 0). Determine:
a) $\vec{a}+\vec{b}$ b) $\overrightarrow{FE}-\overrightarrow{CA}$

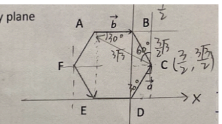
$\vec{a}=(\frac{3}{2},\frac{3\sqrt{3}}{2})$

$\vec{b}=(3,0)$

$\vec{a}+\vec{b}=(\frac{9}{2},\frac{3}{2}\sqrt{3})$

$=(\frac{3}{2},-\frac{3}{2}\sqrt{3})-(-\frac{9}{2},\frac{3}{2}\sqrt{3})$

$=(6,-3\sqrt{3})$.



$\frac{x}{3}=\sin 30°$

$x=3\sin 30°$

$x=\frac{3}{2}$

$y=\frac{3\sqrt{3}}{2}$

#2

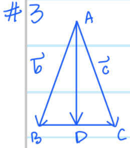
If $\vec{a}=(k,\frac{1}{2},k)$ if a unit vector, determine all possible values of k.

$2k^2+\frac{1}{4}=1$

$2k^2=\frac{3}{4}$ $k^2=\frac{3}{8}$

$k=\pm\sqrt{\frac{3}{8}}=\pm\frac{\sqrt{3}\sqrt{2}}{2\sqrt{2}}=\pm\frac{\sqrt{6}}{4}$.

#3



① $\overrightarrow{AD}=\vec{b}+\frac{1}{2}\overrightarrow{BD}$

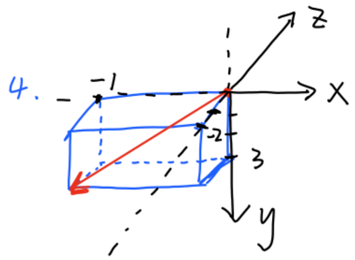
② $\overrightarrow{AD}=\vec{c}+\frac{1}{2}\overrightarrow{DC}$

① + ② : $2\overrightarrow{AD}=\vec{b}+\vec{c}$

$\overrightarrow{AD}=\frac{1}{2}\vec{b}+\frac{1}{2}\vec{c}$

6. Prove that if or not the points A $(3,0,4)$, B $(1,2,5)$, and C $(2,1,3)$ can construct a triangle? If yes, what kind of triangle is it?

4.



$|\overrightarrow{OM}|=\sqrt{(-1)^2+(3)^2+(-2)^2}=\sqrt{14}$

the angle between OM and x-axis : α

y-axis : β

z-axis : γ

$\alpha=\cos^{-1}\frac{-1}{\sqrt{14}}=105.5°$

$\beta=\cos^{-1}\frac{3}{\sqrt{14}}=36.7°$

$\gamma=\cos^{-1}\frac{-2}{\sqrt{14}}=122.3°$

5. find $\vec{r}=\lambda\vec{p}+u\vec{q}$

$(16,11,-24)=\lambda(-2,3,4)+u(4,1,-6)$

$$\begin{cases} 16=-2\lambda+4u & ① \\ 11=3\lambda+u & ② \\ -24=4\lambda-6u & ③ \end{cases}$$

From ① and ② : $u=3$

$\lambda=2$

check it with ③ : $-24\neq 4(2)-6(3)$

$-24\neq 8-18$

$-24\neq -10$

∴ The significance is $\vec{r},\vec{p},\vec{q}$ are not spanning a plane (or not linear dependent) as no such λ and u exist

6. prove whether A, B, C are co-linear by doing $\overrightarrow{AB}=\lambda\overrightarrow{BC}$, if there is no such λ exist, then calculate $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$, $|\overrightarrow{CA}|$