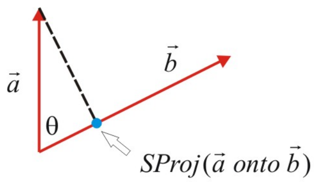
7.5 Scalar and Vector Projections

A Scalar Projection

The *scalar projection* of the vector $\vec{a}$ onto the vector $\vec{b}$ is a *scalar* defined as:

$$SProj(\vec{a} \text{ onto } \vec{b}) = \|\vec{a}\| \cos\theta \quad \text{where } \theta = \angle(\vec{a}, \vec{b})$$



Ex 1. Given two vectors with the magnitudes $\|\vec{a}\| = 10$ and $\|\vec{b}\| = 16$ respectively, and the angle between them equal to $\theta = 120°$, find the scalar projection

a) of the vector $\vec{a}$ onto the vector $\vec{b}$

b) of the vector $\vec{b}$ onto the vector $\vec{a}$

B Special Cases

Consider two vectors $\vec{a}$ and $\vec{b}$.

a) If $\vec{a} \uparrow\uparrow \vec{b}$ ($\cos\theta = 1$), then $SProj(\vec{a} \text{ onto } \vec{b}) = \|\vec{a}\|$

b) If $\vec{a} \uparrow\downarrow \vec{b}$ ($\cos\theta = -1$), then $SProj(\vec{a} \text{ onto } \vec{b}) = -\|\vec{a}\|$

c) If $\vec{a} \perp \vec{b}$ then $SProj(\vec{a} \text{ onto } \vec{b}) = 0$

Ex 2. Find the scalar projection of the vector $\vec{a}$ onto:

a) itself

b) the opposite vector $-\vec{a}$

C Dot Product and Scalar Projection

Recall that the *dot product* of the vectors $\vec{a}$ and $\vec{b}$ is defined as:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$

So, the *scalar projection* of the vector $\vec{a}$ onto the vector $\vec{b}$ can be written as:

$$SProj(\vec{a} \text{ onto } \vec{b}) = \|\vec{a}\| \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

Note:
For a Cartesian (Rectangular) coordinate system, the *scalar components* a_x, a_y, and a_z of a vector $\vec{a} = (a_x, a_y, a_z)$ are the *scalar projections* of the vector $\vec{a}$ onto the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$.

Proof:

$$SProj(\vec{a} \text{ onto } \vec{i}) = \frac{\vec{a} \cdot \vec{i}}{\|\vec{i}\|} = \frac{(a_x, a_y, a_z) \cdot (1,0,0)}{1} = a_x$$

Ex 3. Given the vector $\vec{a} = (2, -3, 4)$, find the scalar projection:

a) of $\vec{a}$ onto the unit vector $\vec{i}$

b) of $\vec{a}$ onto the vector $\vec{i} - \vec{j}$

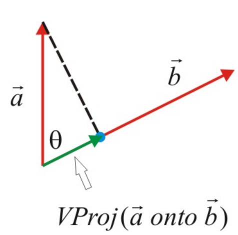
c) of $\vec{a}$ onto the vector $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$

d) of the unit vector $\vec{i}$ onto the vector $\vec{a}$

D Vector Projection

The *vector projection* of the vector $\vec{a}$ onto the vector $\vec{b}$ is a vector defined as:

$$VProj(\vec{a} \text{ onto } \vec{b}) = \|\vec{a}\| \cos\theta \frac{\vec{b}}{\|\vec{b}\|}$$



E Dot Product and Vector Projection

The *vector projection* of the vector $\vec{a}$ onto the vector $\vec{b}$ can be written using the dot product as:

$$VProj(\vec{a} \text{ onto } \vec{b}) = \frac{(\vec{a} \cdot \vec{b})\vec{b}}{\|\vec{b}\|^2}$$

Note:
For a Cartesian (Rectangular) coordinate system, the *vector components* $\vec{a}_x = a_x\vec{i}$, $\vec{a}_y = a_y\vec{j}$, and $\vec{a}_z = a_z\vec{k}$ of a vector $\vec{a} = (a_x, a_y, a_z)$ are the *vector projections* of the vector $\vec{a}$ onto the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$.

Ex 4. Given two vectors $\vec{a} = (0,1,-2)$ and $\vec{b} = (-1,0,3)$, find: a) the vector projection of the vector $\vec{a}$ onto the vector $\vec{b}$ b) the vector projection of the vector $\vec{b}$ onto the vector $\vec{a}$ c) the vector projection of the vector $\vec{a}$ onto the unit vector $\vec{k}$ d) the vector projection of the vector $\vec{i}$ onto the vector $\vec{a}$	Ex 5. Find an expression using the dot product of the vector components of the vector $\vec{a}$ along to the vector $\vec{b}$ and along to a direction perpendicular to the direction of the vector $\vec{b}$ but in the same plan containing the vectors $\vec{a}$ and $\vec{b}$.

Reading: Nelson Textbook, Pages 390-398
Homework: Nelson Textbook: Page 399 # 6, 11, 13, 14