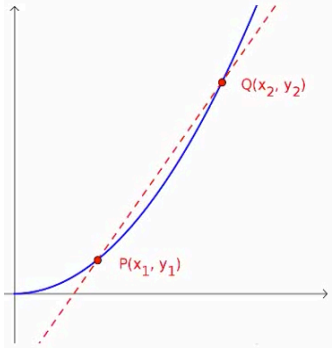
Unit 1 – Functions

Chapter 2.1 – 2.2: Average and instantaneous rate of change

Rates of change $\begin{cases} Average\ rate\ of\ change \\ Intantaneous\ rate\ of\ change \end{cases}$

Given the graph of a function, the **average rate of change** is defined as the slope of the secant line between two points.



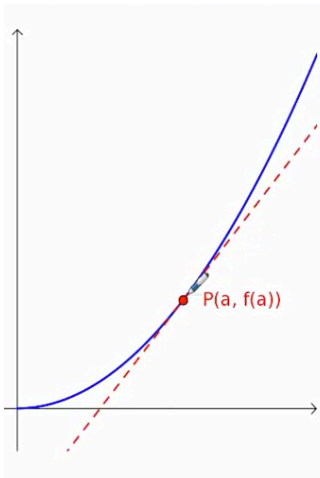
$$\begin{aligned} \text{avg RoC} &= m_{PQ} \\ &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned}$$

Using function notation, the points can also be written:

$$P(x_1, f(x_1))$$
$$Q(x_2, f(x_2))$$

Average RoC = $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The **instantaneous rate of change** is the slope of the tangent line at a particular point of interest, defined by a specified value of the independent variable (e.g., at x = a).



For now, we can only estimate this value by determining the average rate of change over a very small interval near x = a.

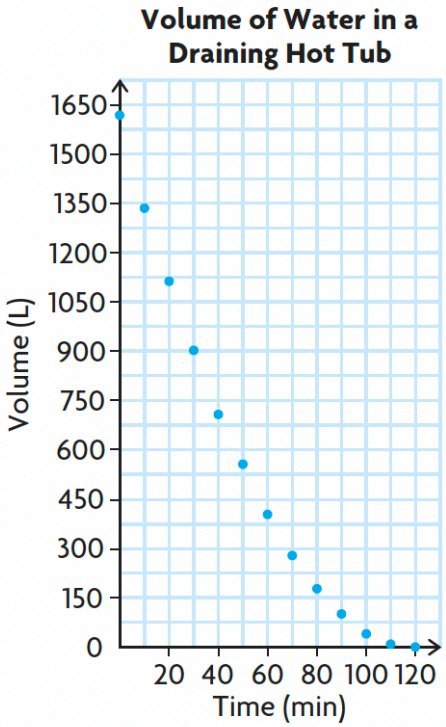
(a) a preceding interval uses a point before the point of interest.

(b) a following interval uses a point after the point of interest.

(c) a centred interval uses points on either side of the point of interest.

Example 1:

Andrew drains the water from a hot tub. The tub holds 1600 L of water. It takes 2 h for the water to drain completely. The volume V, in litres, of water remaining in the tub at various times t, in minutes, is shown in the table and graph.



Time (min)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	10
120	0

Example 2: A bacterial colony starts with 1000 bacteria and doubles each hour.

a) Estimate the growth rate (bacteria/hour) after 2 hours using 1 hour intervals:
   i) Preceding
   ii) Following
   iii) Centered
b) Improve the estimate using 0.1 hour intervals

Example 3: Use the idea of instantaneous rate of change to prove the vertex of $y = 2(x-2)^2 + 1$ is a minimum.

In general, we algebraically represent the estimated instantaneous rate of change as a difference quotient.

For $x = a$, the point of interest is $P(a, f(a))$

The following point occurs at $x = a + h$, where h is an aribitrarily small value, giving a second point $Q(a + h, f(a + h))$

$$\begin{aligned} \text{RoC} &= m_{PQ} \\ &= \frac{f(a+h)-f(a)}{(a+h)-a} \\ &= \frac{f(a+h)-f(a)}{h} \end{aligned}$$

To estimate instantaneous rate of change:

a) Use a series of preceding and/or following intervals, keeping the point of interest constant. As the intervals get smaller and smaller, look for the trend in values.

b) Use a series of centered intervals and look for the trend.

c) Use the difference quotient for very small values of h (both positive and negative work).

The Best estimates come from the smallest intervals.

Suggested questions from Textbook:
Pg76/ #8, 9
Pg 85 / #4, 7, 9, 10, 15