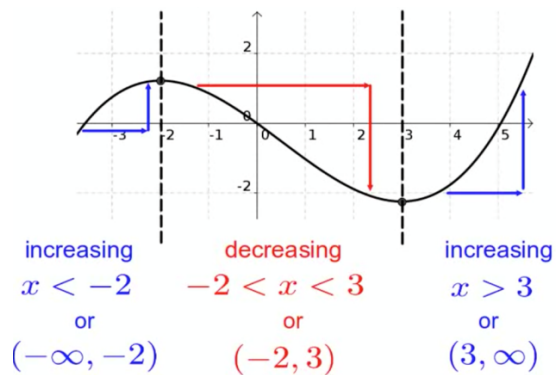




**Unit 2 – Polynomials**  
**Review**

There are 7 properties/key features of functions which can be used to classify and compare them between each other:

- 1) Domain and range
- 2) x-intercepts (zeros) and y-intercept
- 3) Location of any discontinuities (e.g., asymptotes, holes), otherwise the function is continuous
- 4) Intervals of increase or decrease (always read graph left-to-right, x-values increasing)

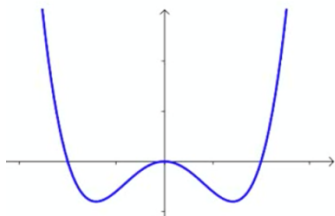


- 5) Turning points occur where functions change from increasing to decreasing, or vice versa  
i.e., local maximum/minimum & absolute maximum/minimum



6) Function symmetry

a) Even symmetry can be seen graphically as a mirror image across the y-axis.



Any point  $(x,y)$  has a corresponding point  $(-x,y)$

Algebraically:

$$f(-x) = f(x)$$

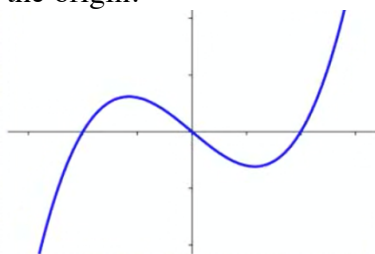
Example: Show that  $f(x) = x^2(x - 2)(x + 2)$ , shown above is an even function.

b) Odd symmetry is more difficult to see in the graph, as it represents a rotational, rather than reflective, symmetry about the origin.

Any point  $(x,y)$  has a corresponding point  $(-x,-y)$ .

Algebraically,

$$f(-x) = -f(x)$$

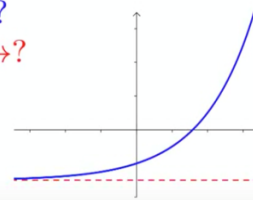


Example: Show that  $g(x) = x(x - 2)(x + 2)$ , shown above is an odd function.

7) End Behavior describes the tendency of the y-values as x-values approach very large positive and negative values (which we express abstractly as infinity)

Example: Describe the end behavior of  $f(x) = 2^x - 3$

as  $x \rightarrow \infty, y \rightarrow ?$   
as  $x \rightarrow -\infty, y \rightarrow ?$





**Chapter 3.1: Polynomial functions**

Consider the familiar functions:

Linear:

Quadratic:

If this pattern continues:

Cubic:

Quartic:

Quintic:

In general, a polynomial function in standard form is:  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$   
where  $\{a_0, a_1, \dots, a_{n-1}, a_n \in R\}$  and  $\{n \in W\}$

Notes:

- 1)  $a_n$  is the leading coefficient
- 2) The degree of a polynomial is the value of the highest exponent
- 3) A polynomial in standard form has descending powers of x.

These are polynomial expressions.	These are not polynomial expressions.
$3x^2 - 5x + 3$	$\sqrt{x} + 5x^3$
$-4x + 5x^7 - 3x^4 + 2$	$\frac{1}{2x + 5}$
$\frac{2}{5}x^3 - 3x^5 + 4$	$6x^3 + 5x^2 - 3x + 2 + 4x^{-1}$
$\sqrt{4}x^3 - \frac{\sqrt{5}}{3}x^2 + 2x - \frac{1}{4}$	$\frac{3x^2 + 5x - 1}{2x^2 + x - 3}$
$3x - 5$	$4^x + 5$
$-7$	$\sin(x - 30)$
$-4x$	$x^2y + 3x - 4y^{-2}$
$(2x - 3)(x + 1)^2$	$3x^3 + 4x^{2.5}$



Recall: **Finite differences**

- 1) First differences are a constant for \_\_\_\_\_.
- 2) Second differences are a constant for \_\_\_\_\_.

Higher-order finite differences can be used to identify other polynomials from data points.

For an Order-N polynomial, the Nth difference are a constant.

**3rd degree**  
 $y = 0.1x^3 - x^2 + 3x - 5$

x	y	$D_1$	$D_2$	$D_3$
-5	-57.5			
0	-5	52.5		
5	-2.5	2.5	-50	75
10	25	27.5	25	75
15	152.5	127.5	100	75
20	455	302.5	175	

Domain is always  $\{x|x \in \mathbb{R}\}$ .

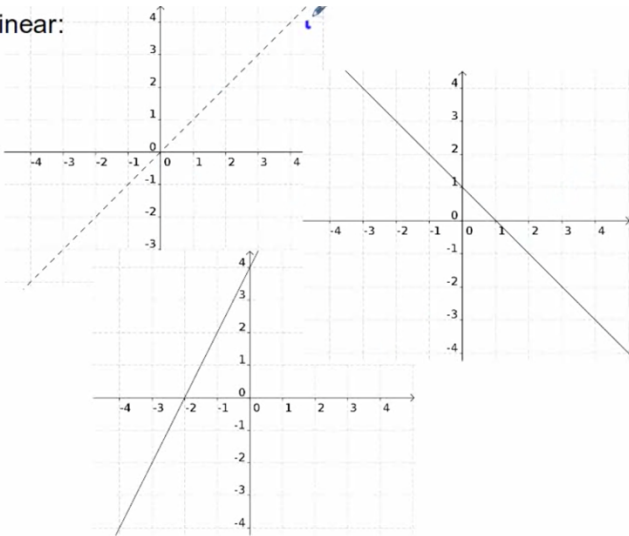


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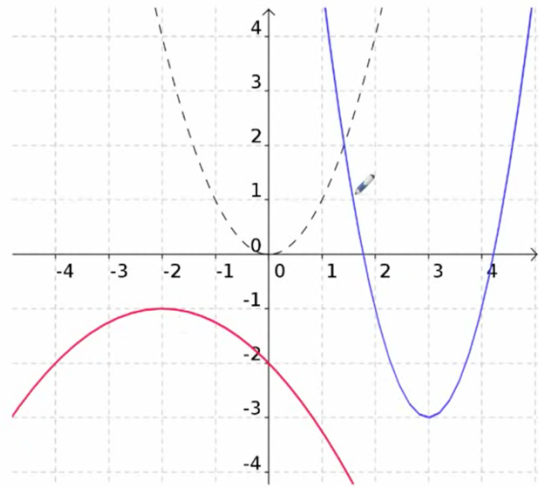
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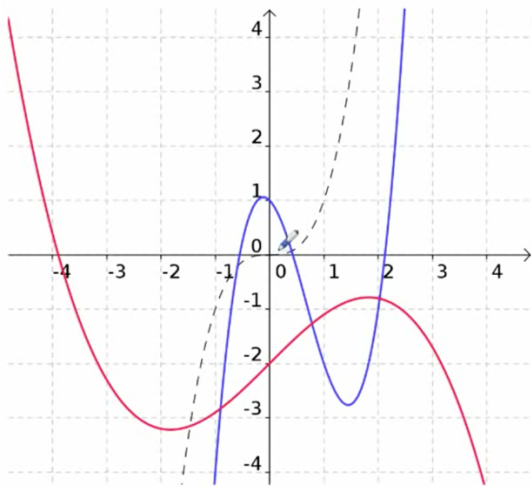
Linear:



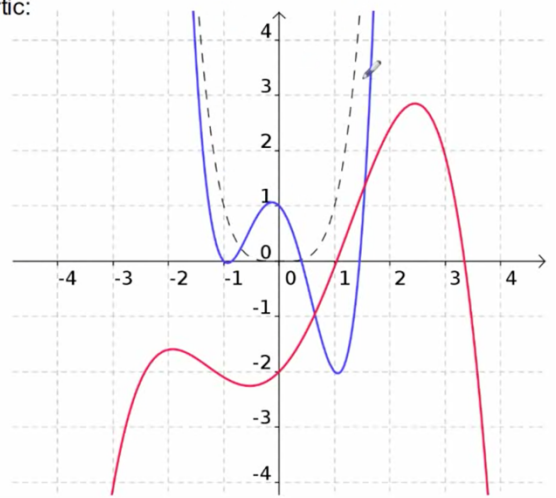
Quadratic:



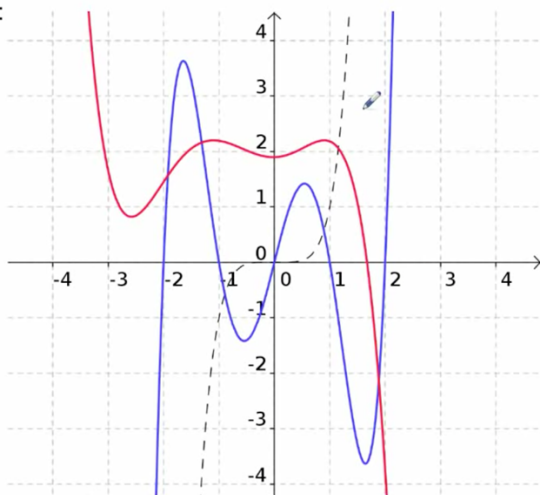
Cubic:



Quartic:



Quintic:

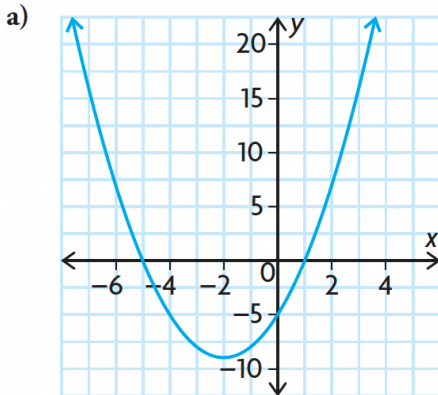




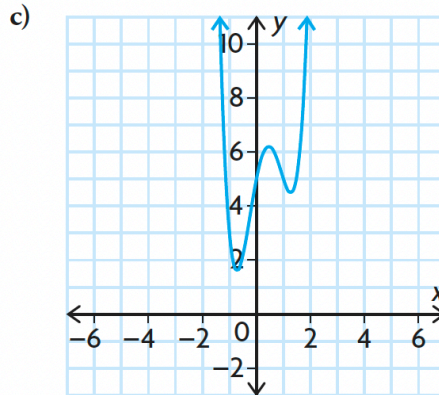
**Investigation:**  
**Chapter 3.2 → Characteristics of polynomial functions**

How can you predict some of the characteristics of a polynomial function from its equation?

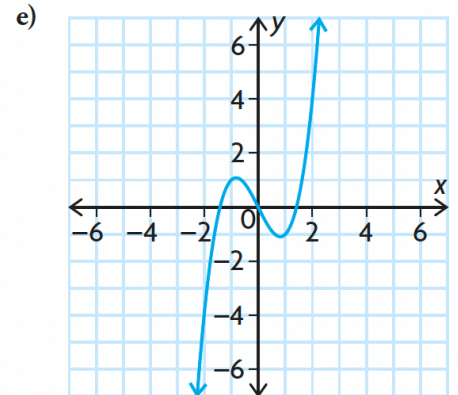
The graphs of some polynomial functions are shown below.



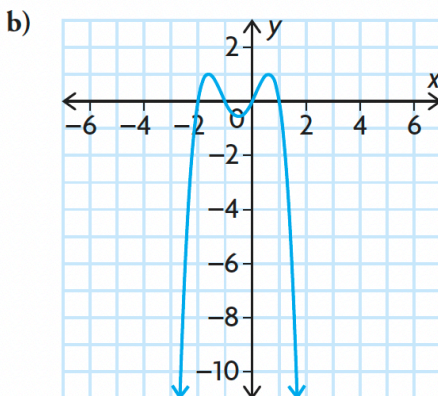
$$f(x) = x^2 + 4x - 5$$



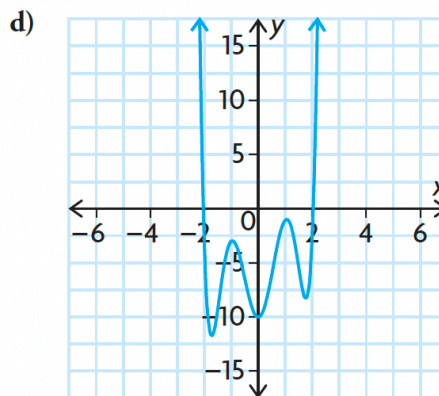
$$f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$$



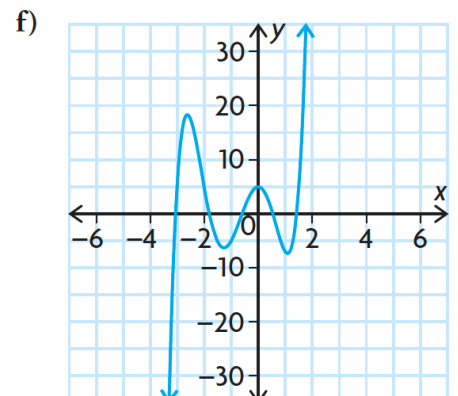
$$f(x) = x^3 - 2x$$



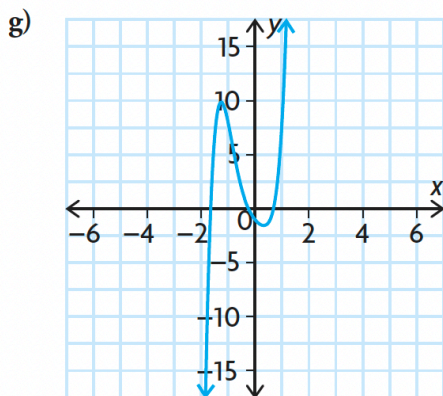
$$f(x) = -x^4 - 2x^3 + x^2 + 2x$$



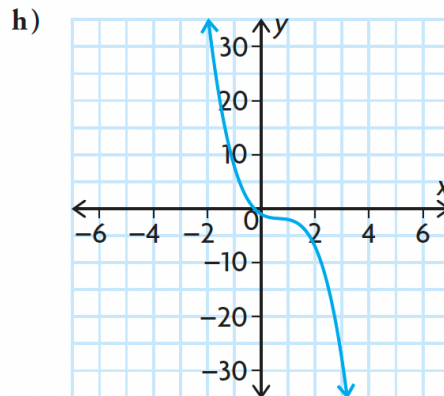
$$P(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$$



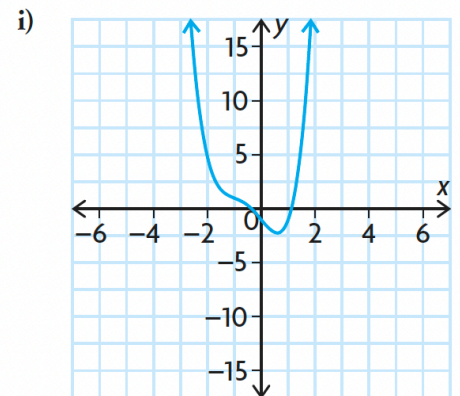
$$f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$$



$$f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$$



$$f(x) = -2x^3 + 4x^2 - 3x - 1$$



$$f(x) = x^4 + 2x^3 - 3x - 1$$



**Part A:** In the following table, complete it using the equations and graphs given above.

Equation & Graph	Degree	Even or Odd degree?	Leading coefficient	End behaviors		Number of Turning points
				$x \rightarrow -\infty$	$x \rightarrow \infty$	
a)	2	even	+1	$y \rightarrow +\infty$	$y \rightarrow +\infty$	1
b)						
c)						
d)						
e)						
f)						
g)						
h)						
i)						

**Part B:** Create two new polynomial functions of degree greater than 2, one of even degree and one of odd degree. Do these new polynomial functions support your observation in part A?

Function 1:

Function 2:

**Part C:** What do you think is the maximum number of turning points that a polynomial function of degree  $n$  can have?



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**Part D:** Graph the following functions using DESMOS (<https://www.desmos.com/calculator>).

Copy each graph roughly and its equation into the appropriate column of a table like the one shown below.

- i)  $f(x) = x^4 - 2x^2 + 1$       vi)  $f(x) = x^5 - 3x$   
ii)  $f(x) = x^3 + 3x^2 - 2x - 5$       vii)  $f(x) = x^2 - 3x + 4$   
iii)  $f(x) = \frac{1}{2}x^{10} - \frac{1}{3}x^4 + x^2$       viii)  $f(x) = 2x^7 - 3x^3 + 2x$   
iv)  $f(x) = x^3 + x$       ix)  $f(x) = -3x^4 + 2x^3 - 3x + 1$   
v)  $f(x) = -2x^6 + 3x^4$       x)  $f(x) = x^2 - x$

Even Functions (symmetry in the y-axis)	Odd functions (rotational symmetry around the origin)	Neither Even nor Odd functions (neither of these symmetries)





**Part E:** Is every function of even degree an even function?

Is every function of odd degree an odd function?

Hint: You could determine that by looking at graphs or algebraically.

i.e., Even function has  $f(-x) = f(x)$ ; Odd function has  $f(-x) = -f(x)$ .

**Part F:** How can you use the equation of a polynomial function to describe its end behaviors, number of turning points, and symmetry?

**Reflecting:**

**Part G:** Why must all polynomial functions of even degree have an absolute maximum or absolute minimum?

**Part H:** Why must all polynomial functions of odd degree have at least one zero?

The end of investigation.



Although the graphs of polynomials can appear very different, they also have many common and predictable properties.

**1) Even/Odd polynomials**

The order or degree is the highest exponents, and this value (even or odd) determines some behaviors.

**2) Leading coefficient**

The sign of the coefficient of the highest order term will determine if the polynomial is reflected, affecting the overall appearance of the graph:

- i) How the graph starts/ends
- ii) End behavior
- iii) Direction of opening (for even polynomials)

**3) Turning points**

A turning point occurs whenever the graph changes from increasing to decreasing, or decreasing to increasing.

**4) Extreme values**

Absolute maximum or minimum values are the greatest or least values for the entire polynomial (i.e., entire domain)

Even polynomials will always have an absolute extrema.

Odd polynomials will never have an absolute extrema.

Local extrema occur at any turning points which are not necessarily absolute extrema.

**5) Number of zeros**

Zeros occur where the graph crosses the x-axis.

Even polynomials may have no zeros, or more.

Odd polynomials must have at least one zero.

**6) End behavior**

Even polynomials have identical behavior at each end.

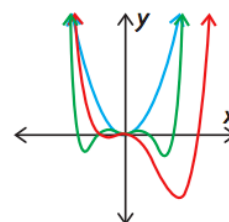
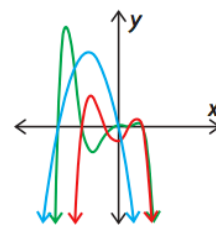
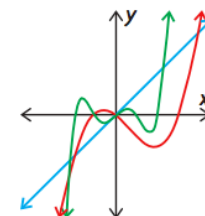
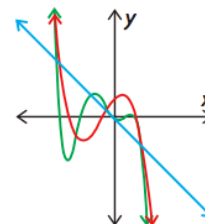
Odd polynomials have opposite behavior at each end.



## Summary:

### End Behaviours

- An odd-degree polynomial function has opposite end behaviours.
  - If the leading coefficient is negative, then the function extends from the second quadrant to the fourth quadrant; that is, as  $x \rightarrow -\infty, y \rightarrow \infty$  and as  $x \rightarrow \infty, y \rightarrow -\infty$ .
  - If the leading coefficient is positive, then the function extends from the third quadrant to the first quadrant; that is, as  $x \rightarrow -\infty, y \rightarrow -\infty$  and as  $x \rightarrow \infty, y \rightarrow \infty$ .
- An even-degree polynomial function has the same end behaviours.
  - If the leading coefficient is negative, then the function extends from the third quadrant to the fourth quadrant; that is, as  $x \rightarrow \pm\infty, y \rightarrow -\infty$ .
  - If the leading coefficient is positive, then the function extends from the second quadrant to the first quadrant; that is, as  $x \rightarrow \pm\infty, y \rightarrow \infty$ .



### Turning Points

- A polynomial function of degree  $n$  has at most  $n - 1$  turning points.

### Number of Zeros

- A polynomial function of degree  $n$  may have up to  $n$  distinct zeros.
- A polynomial function of odd degree must have at least one zero.
- A polynomial function of even degree may have no zeros.

### Symmetry

- Some polynomial functions are symmetrical in the  $y$ -axis. These are even functions, where  $f(-x) = f(x)$ .
- Some polynomial functions have rotational symmetry about the origin. These are odd functions, where  $f(-x) = -f(x)$ .
- Most polynomial functions have no symmetrical properties. These are functions that are neither even nor odd, with no relationship between  $f(-x)$  and  $f(x)$ .



Example: Reasoning about characteristics of a given polynomial function

Describe the end behaviours of each function, the possible number of turning points, and the possible number of zeros. Use these characteristics to sketch possible graphs of the function.

a)  $f(x) = -3x^5 + 4x^3 - 8x^2 + 7x - 5$       b)  $g(x) = 2x^4 + x^2 + 2$

**Solution**

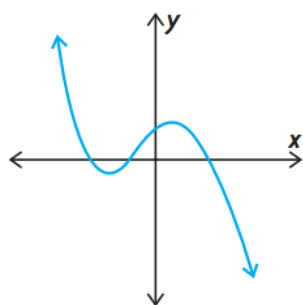
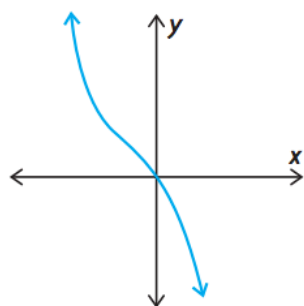
a)  $f(x) = -3x^5 + 4x^3 - 8x^2 + 7x - 5$

The degree is odd, so the function has opposite end behaviours. The leading coefficient is negative, so the graph must extend from the second quadrant to the fourth quadrant.

As  $x \rightarrow -\infty, y \rightarrow +\infty$ .

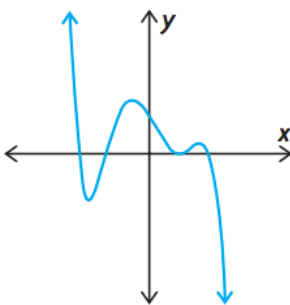
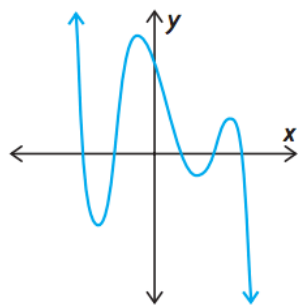
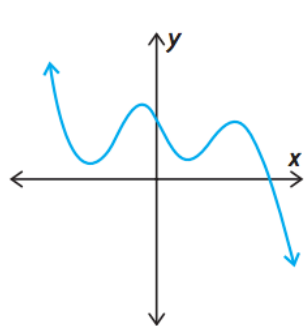
As  $x \rightarrow +\infty, y \rightarrow -\infty$ .

If  $x$  is a very large negative number, such as  $-1000$ ,  $-3x^5$  will have a large positive value and will have a greater effect on the value of the function than the other terms. Therefore, the graph will pass through the second quadrant. For very large positive values of  $x$ ,  $-3x^5$  will have a large negative value. Therefore, the graph will extend into the fourth quadrant.



Using the end behaviours of the function, sketch possible graphs of a fifth-degree polynomial.  
To pass through the second quadrant and extend into the fourth quadrant, the graph must have an even number of turning points.

$f(x)$  may have zero, two, or four turning points.



Since the function is a fifth-degree polynomial, it must have at least one zero and no more than five zeros.



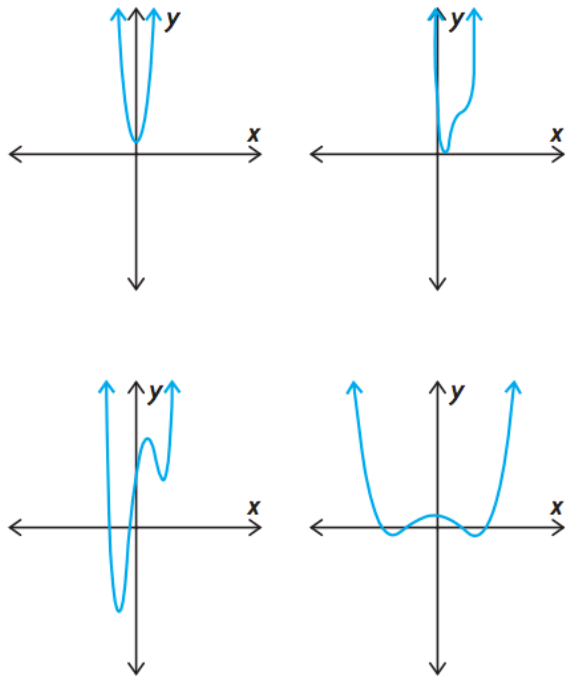
b)  $g(x) = 2x^4 + x^2 + 2$

The degree is even, so the function has the same end behaviours. The leading coefficient is positive, so the graph must extend from the second quadrant to the first quadrant.

As  $x \rightarrow -\infty, y \rightarrow +\infty$ .

As  $x \rightarrow +\infty, y \rightarrow +\infty$ .

If  $x$  is a very large negative number,  $2x^4$  will have a large positive value and will have a greater effect on the value of the function than the other terms. Therefore, the graph will pass through the second quadrant. For very large positive values of  $x$ ,  $2x^4$  will have a large positive value. Therefore, the graph will extend into the first quadrant.



Using the end behaviours of the function, sketch possible graphs of a fourth-degree polynomial.

To start in the second quadrant and end in the first quadrant, the graph must have an odd number of turning points.

Since the function is a fourth-degree polynomial, it may have anywhere from zero to four  $x$ -intercepts.

$f(x)$  may have one or three turning points and zero, one, two, three, or four zeros.

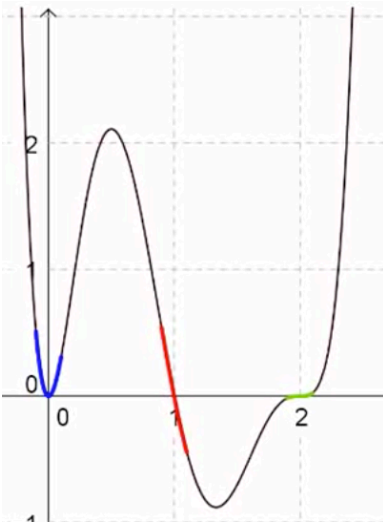


**Unit 2 – Polynomials**

**Chapter 3.3: Characteristics of Polynomial functions in Factored form**

Example 1: The order of the factors determine the behavior of the graph near the x-axis.

Consider  $f(x) = 5x^2(x - 1)(x - 2)^3$





## In Summary

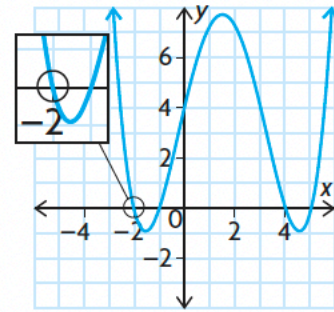
### Key Idea

- The zeros of the polynomial function  $y = f(x)$  are the same as the roots of the related polynomial equation,  $f(x) = 0$ .

### Need to Know

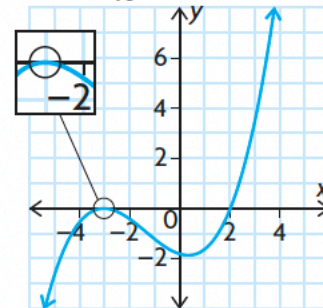
- To determine the equation of a polynomial function in factored form, follow these steps:
  - Substitute the zeros  $(x_1, x_2, \dots, x_n)$  into the general equation of the appropriate family of polynomial functions of the form  $y = a(x - x_1)(x - x_2) \dots (x - x_n)$ .
  - Substitute the coordinates of an additional point for  $x$  and  $y$ , and solve for  $a$  to determine the equation.
- If any of the factors of a polynomial function are linear, then the corresponding  $x$ -intercept is a point where the curve passes through the  $x$ -axis. The graph has a linear shape near this  $x$ -intercept.

$$y = \frac{1}{10}(x + 2)(x + 1)(x - 4)(x - 5)$$



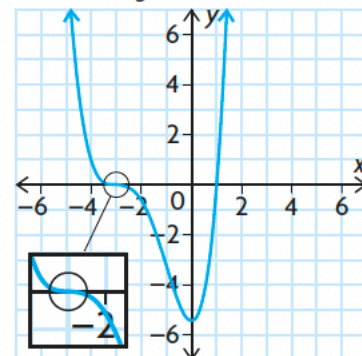
- If any of the factors of a polynomial function are squared, then the corresponding  $x$ -intercepts are turning points of the curve and the  $x$ -axis is tangent to the curve at these points. The graph has a parabolic shape near these  $x$ -intercepts.

$$y = \frac{1}{10}(x + 3)^2(x - 2)$$



- If any of the factors of a polynomial function are cubed, then the corresponding  $x$ -intercepts are points where the  $x$ -axis is tangent to the curve and also passes through the  $x$ -axis. The graph has a cubic shape near these  $x$ -intercepts.

$$y = \frac{1}{5}(x + 3)^3(x - 1)$$

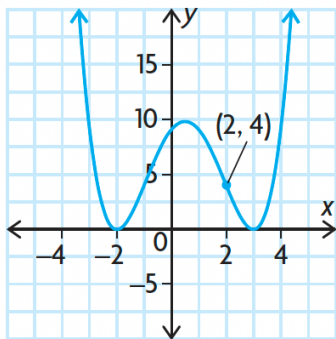




Example 2: Sketch a possible graph of the function  $f(x) = -(x + 2)(x - 1)(x - 3)^2$ .

Example 3: Write the equation of a cubic function that has zeros at  $-2$ ,  $3$ , and  $\frac{2}{5}$ . The function also has a y-intercept of  $6$ .

Example 4: Write the equation of the function shown below. And state the domain and range of the function.



Example 5: Sketch the graph of  $f(x) = x^4 + 2x^3$ .





**Unit 2 – Polynomials**  
**Chapter 3.5: Long Division**

Recall: How to do the long division of  $107 \div 4$  ?

$$\begin{array}{r} 26 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 4 \overline{)107} \leftarrow \text{dividend} \\ \underline{8} \phantom{0} \\ 27 \\ \underline{24} \\ 3 \leftarrow \text{remainder} \end{array}$$

Every division statement that involves numbers can be rewritten using multiplication and addition. The multiplication is the quotient, and the addition is the remainder. For example, since  $107 = (4)(26) + 3$ , then  $\frac{107}{4} = 26 + \frac{3}{4}$ . The quotient is 26, and the remainder is 3.

**Example 1:** Use **long division** and **synthetic division** to determine the quotient and remainder for  $(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$



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**Example 2:** Use long division to determine the remainder of  $\frac{5x-2x^3+3+x^4}{1+2x+x^2}$ .

**Example 3:** Use synthetic division to determine whether  $x + 2$  is a factor of  $13x - 2x^3 + x^4 - 6$ .



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**Example 4:**  $2x + 3$  is one factor of the function  $f(x) = 6x^3 + 5x^2 - 16x - 15$ . Determine the other factors. Then determine the zeros, and sketch a graph of the polynomial.



**Unit 2 – Polynomials**

**Chapter 3.6: Factoring Theorem and Remainder Theorem**

**Remainder Theorem:** When a polynomial,  $f(x)$ , is divided by  $x - a$ , the remainder is equal to  $f(a)$ .

For example:  $\frac{3x^3 - 5x^2 - 7x - 1}{x - 3} = 3x^2 + 4x + 5 + \frac{14}{x - 3}$ , where the remainder is 14.

**OR**

The remainder of  $3x^3 - 5x^2 - 7x - 1$  divided by  $x - 3$  can be obtained from evaluating  $f(3) = 14$

**Factor Theorem:** If the remainder, or  $f(a)$ , is equal to zero, then  $x - a$  is a factor of the polynomial  $f(x)$ .

**Example 1:** Use remainder theorem to determine the remainder when  $x^3 + 7x^2 + 2x - 5$  is divided by  $x + 7$ .



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**Example 2:** Use factor theorem to factor  $x^3 - 5x^2 - 2x + 24$

**Practice 1:** Use the factor theorem to determine factors of  $f(x) = x^3 + 4x^2 + x - 6$ , then sketch.



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**Practice 2:** Sketch a graph of the function  $y = 4x^4 + 6x^3 - 6x^2 - 4x$

**Practice 3:** Use grouping method to factor  $x^4 - 6x^3 + 2x^2 - 12x$



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**Example 3:** When  $2x^3 - mx^2 + nx - 2$  is divided by  $x + 1$ , the remainder is  $-12$  and  $x - 2$  is a factor.  
Determine the values of  $m$  and  $n$ .



## Unit 2 – Polynomials

### Chapter 3.7: Factoring Sum and Difference of Cubes

Recall: **Perfect squares & Difference of Squares**

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

**Sum and difference of cubes:**

- An expression that contains two perfect cubes that are added together is called a sum of cubes and can be factored as follows:

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

- An expression that contains perfect cubes where one is subtracted from the other is called a difference of cubes and can be factored as follows:

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

With higher-order polynomials, look for patterns involving squares or cubes.

For example:

$$a^4 - b^4 = (a^2)^2 - (b^2)^2 =$$

$$x^{12} + y^{12} = (x^3)^4 + (y^3)^4 \text{ or } (x^6)^2 + (y^6)^2 \text{ or } (x^4)^3 + (y^4)^3 =$$





**Example 1:** Factor  $64x^6 - 729$

**Example 2:**  $(x - 3)^3 + (3x - 2)^3$

**Example 3:**  $x^6 - \frac{1}{64}y^{12}$



## Unit 2 – Polynomials

### Chapter 3.4: Transformations of Polynomials

Any function,  $f(x)$ , can be transformed to  $y = af[k(x - d)] + c$

This is most easily accomplished by transforming key points of features of the parent function:

$$(x, y) \rightarrow \left( \frac{x}{k} + d, ay + c \right)$$

Polynomials:  $y = a[k(x - d)]^n + c$

## In Summary

### Key Ideas

- You can graph functions of the form  $g(x) = af[k(x - d)] + c$  by applying the appropriate transformations to the key points of the parent function, one at a time, making sure to apply  $a$  and  $k$  before  $c$  and  $d$ . This order is like the order of operations for numerical expressions, since multiplications (stretches, compressions, and reflections) are done before additions and subtractions (translations).
- When using transformations to graph, you can apply  $a$  and  $k$  together, then  $c$  and  $d$  together, to get the desired graph in fewer steps.

### Need to Know

- The value of  $a$  determines the vertical stretch or compression and whether there is a reflection in the  $x$ -axis:
  - When  $|a| > 1$ , the graph of  $y = f(x)$  is stretched vertically by the factor  $|a|$ .
  - For  $0 < |a| < 1$ , the graph is compressed vertically by the factor  $|a|$ .
  - When  $a < 0$ , the graph is also reflected in the  $x$ -axis.
- The value of  $k$  determines the horizontal stretch or compression and whether there is a reflection in the  $y$ -axis:
  - When  $|k| > 1$ , the graph is compressed horizontally by the factor  $\frac{1}{|k|}$ .
  - When  $0 < |k| < 1$ , the graph is stretched horizontally by the factor  $\frac{1}{|k|}$ .
  - When  $k < 0$ , the graph is also reflected in the  $y$ -axis.
- The value of  $d$  determines the horizontal translation:
  - For  $d > 0$ , the graph is translated  $d$  units right.
  - For  $d < 0$ , the graph is translated  $d$  units left.
- The value of  $c$  determines the vertical translation:
  - For  $c > 0$ , the graph is translated  $c$  units up.
  - For  $c < 0$ , the graph is translated  $c$  units down.



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**Unit 2 – Solving equations and inequalities**

**Chapter 4.1: Solving polynomial equations and word problems**

Example 1: Solve  $3x^3 + 8x^2 = -3x + 2$

Example 2: Determine the exact roots of  $x^3 - 4x^2 + 2x + 3$

Step 1: rewrite the equation of it to equal to zero.

Step 2: define the resulting polynomial as a function and apply the factor theorem.

Step 3: factor out the first term, and repeat until in a fully factored form.

Step 4: find the roots of the equation (i.e., set it back to zero and solve)

Step 5: ignore solutions that are outside of the domain defined by the conditions of the problem.

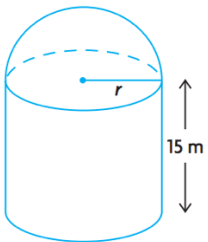


Example 3: A box is in the shape of a rectangular prism. One side is a square, and the length is 12 units longer than the square sides. The volume of the box is 135 cubic units. What are the dimensions of the box?

Practice 1 :

Amelia's family is planning to build another silo for grain storage, identical to those they have on their farm. The cylindrical portion of those they currently have is 15 m tall, and the silo's total volume is  $684\pi \text{ m}^3$ .

Determine possible values for the radius of the silo.





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### Independent practice 2:



The paths of two orcas playing in the ocean were recorded by some oceanographers. The first orca's path could be modelled by the equation  $h(t) = 2t^4 - 17t^3 + 27t^2 - 252t + 232$ , and the second by  $h(t) = 20t^3 - 200t^2 + 300t - 200$ , where  $h$  is their height above/below the water's surface in centimetres and  $t$  is the time during the first 8 s of play. Over this 8-second period, at what times were the two orcas at the same height or depth?



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### Unit 2 – Solving equations and inequalities

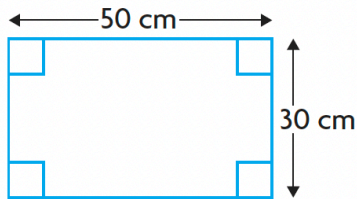
#### Chapter 4.2 – 4.3: Solving linear inequalities and polynomial inequalities

Example 1: Solve  $x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$  (Hint: factoring by grouping)



Example 2:

An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of such a box. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than  $4000 \text{ cm}^3$ .







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Independent practice:

The elevation of a hiking trail is modelled by the function

$h(x) = 2x^3 + 3x^2 - 17x + 12$ , where  $h$  is the height measured in metres above sea level and  $x$  is the horizontal position from a ranger station measured in kilometres. If  $x$  is negative, the position is to the west of the station, and if  $x$  is positive, the position is to the east. Since the trail extends 4.2 km to the west of the ranger station and 4 km to the east, the model is accurate where  $x \in [-4.2, 4]$ .

How can we determine which sections of the trail are above the sea level?