

## Formulas

### Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

### Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

### Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

### Addition and Subtraction Formulas

$$\sin (x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin (x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos (x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos (x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan (x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

**NO CALCULATOR!!!!**

### Self-Assessment:

**Total mark: \_\_\_\_\_/36**

1. (4 marks) Express each of the following as a single trigonometric ratio first, then, evaluate.

a)  $1 - 2\sin^2 \frac{7\pi}{12}$

b)  $\frac{2\tan 157.5^\circ}{1 - \tan^2 157.5^\circ}$

2. (2 marks) Write two equivalent expressions for  $\sec \frac{9\pi}{10}$ , one use related acute angle, another use co-function identities.

$$\sec \frac{9\pi}{10} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

3. (4 marks) Solve the equation.  $2\sin^2x - 7\sin x + 3 = 0$ ,  $0 \leq x \leq 2\pi$ .
4. (4 marks) Determine the exact value of  $\cos \frac{5\pi}{12}$ . Fully simplify, evaluate, and rationalize if necessary.
5. (4 marks) Expand and simplify first, then determine the exact value of  $(\sin \frac{7\pi}{12} + \cos \frac{7\pi}{12})^2$ .
6. (4 marks) Determine  $\sin x[2 \cos(2x)] + \sin x = 0$ , on the interval  $0 \leq x \leq 2\pi$ .
7. (4 marks) Determine the exact value of  $\sin \frac{\theta}{2}$ , given that  $\theta$  is in quadrant 4 and  $\tan \theta = -\frac{12}{5}$ .

8. (4 marks) If  $\tan x = -1$  and  $\cos y = \frac{4}{5}$  with  $x$  and  $y$  in the interval  $[\frac{3\pi}{2}, 2\pi]$ , determine the exact value of  $\csc(x - y)$ . Fully simplify and rationalize, if necessary.

9. (6 marks) Prove identities.

a)  $\sec 2x = \frac{\csc x}{\csc x - 2\sin x}$

b)  $\frac{\sin 2x}{1 - \cos 2x} = \cot x$