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MCR3U Unit 2: Rational and Radicals
Lesson 1: adding, subtracting, and multiplying polynomials

Learning Goals:
I will be able to:

- ✓ Add and subtract polynomials by combining like terms
- ✓ Compare two polynomial functions or expressions for equivalency
- ✓ Multiply polynomials by applying the commutative, distributive, and associative properties

Steps:

1) Place like terms together
2) Add the like terms

Example 1:
$(12x^2 + 2x + 7) + (2x^2 - x + 1)$

Example 2:
$(m^2 - 2n^2 + n - 1) - (m + 2n^2 + n - 3)$

Example 3:
$(-xy + xz + yz) - (xy + 2xz) + (2xy + 4xz - 4yz)$

Determining equivalency

Two polynomial functions equivalent if:

1) They simplify algebraically to give the same function
2) They make the same graph

Two polynomial functions not equivalent if:

1) They result in different values when evaluated with the same number

Example 4: Determine whether the two functions are equivalent:

a) $f(x) = (3x^2 + 7x) - (2x^2 + 6x - 1)$ and $g(x) = (5x^2 + 2) + (-4x^2 + x - 1)$

b) $h(t) = t(t + 1) - t^2$ and $g(t) = t^2(1 - t) - (t + 2)$

c) $y_1 = (x + 3)(x) - (x - 1)$ and $y_2 = x^2(x + 3) - (x^3 - 1)$

Practice:
Textbook pg. 88-89 # 6, 7, 8adf, 11, 13, 17

Multiplying polynomials
For any polynomials a, b, and c:

$ab = ba$ Commutative property

$(ab)c = a(bc)$ Associative property

$a(b + c) = ab + ac$ Distributive property

Example 5:
$(x^2 + 5x + 1)(3x^2 - 10x + 15)$

Example 6:
Let an integer be represented by x. Find, in terms of x, the product of 4 consecutive integers starting with x.

Practice:
Textbook pg. 95-96 # 4, 5, 9, 13

MCR3U Unit 2: Rational and Radicals
Lesson 2: Simplifying, multiplying, and dividing rationals

Learning Goals:
I will be able to:

- ✓ **Simplify rational expressions and functions and state the restrictions**
- ✓ **Multiply and divide rational expressions and state the restrictions.**

A rational function is the ratio of two polynomial functions:

$$f(x) = \frac{P(x)}{Q(x)}, Q(x) \neq 0$$

e.g. $f(x) = \frac{x^2-2x+3}{4x-1}$ or $g(x) = \frac{x^3-x^2+10}{x^2-1}$

Example 1:

Simplify and state any restrictions on the variables.
(Restrictions come from the factored expression before simplifying/cancelling out)

$$\frac{3x^4y^4 + 9x^2y^2}{15x^3y^3}$$

Example 2:

Simplify the function and state the domain

$$f(v) = \frac{v - 5}{v^2 - 25}$$

$$f(x) = \frac{2x^4 - 28x^3 + 98x^2}{x^4 - 4x^3 - 21x^2}$$

Multiplying and dividing rational expressions

Same rules as for multiplying and dividing rational numbers:

$$\frac{P}{Q} \times \frac{R}{S} = \frac{PR}{QS}, \quad Q, S \neq 0$$

$$\frac{p}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R} = \frac{PS}{QR}, \quad Q, S, R \neq 0$$

Example 3:

Simplify and state the restrictions

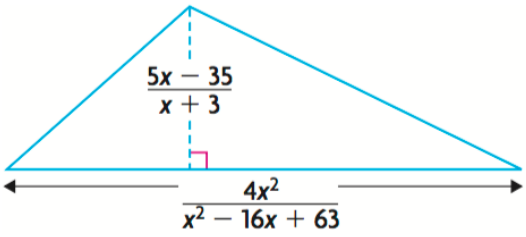
$$\frac{14y^2}{3x^2} \times \frac{2x^4y^3}{7x^2y^2}$$

$$\frac{6x+3}{3x} \times \frac{x^2}{2x+1}$$

$$\frac{2x+10}{x^2-4x+4} \div \frac{x^2-25}{x-2}$$

Example 4:

Determine the area of the triangle in simplified form state the restriction.



Example 8:
Simplify. State restrictions on the variables.

$$\frac{\dfrac{m^2 - mn}{6m^2 + 11mn + 3n^2} \div \dfrac{m^2 - n^2}{2m^2 - mn - 6n^2}}{\dfrac{4m^2 - 7mn - 2n^2}{3m^2 + 7mn + 2n^2}}$$

Practice:
Textbook pg. 113 #4, 5, 7, 8, 9 & pg. 122-123 #5, 6, 7, 8, 10, 12

MCR3U Unit 2: Rational and Radicals
Lesson 3: Simplifying, adding, and subtracting rationals

Learning Goals:
I will be able to:

- ✓ **Simplify rational expressions and functions and state the restrictions**
- ✓ **Adding and subtracting rational expressions and state the restrictions.**

Need to know:

- To add or subtract rational functions or expressions, determine the lowest common denominator (LCD). To do this, factor all the denominators. The LCD consists of the product of any common factors and all the unique factors.
- The LCD is not always the product of all the denominators.
- After finding the LCD, rewrite each term using the LCD as the denominator and then add or subtract numerators.
- Restrictions are found by finding the zeros of all denominators, that is, the zeros of the LCD.

Example 1: Simplify and state any restrictions on the variables:

$$\frac{3n}{2n+1}+\frac{4}{n-3}$$

$$\frac{2t}{t^2-1}-\frac{t+2}{t^2+3t-4}$$

Let's take a look at Example 1 on Textbook pg124 together before doing the practice.

Practice:
Textbook pg. 128 #5a) c), 6 e)f), 7d)e)f), 8, 9c) d), 11.

MCR3U Unit 2: Rational and Radicals
Lesson 4: Operations with radicals

Learning Goals:
I will be able to:

- ✓ **Perform operations on radical expressions**
- ✓ **Simplify radical expressions by rationalizing the denominator**

Rules and properties: square root expressions in simplest form

Parts of radical
$\sqrt[3]{27}$
3 is index/coefficient; 27 is radicand; and take it as a whole is called radical.

An expression involving square roots is in simplest form if

1. there are no perfect-square factors in a radical.
2. No fraction appears inside a radical.
3. No radical appears in the denominator.

What are the rules?

$7\sqrt{5}+1\sqrt{5}=8\sqrt{5}$	1) "Like terms" or "like radicals" $\rightarrow$ $a\sqrt{b}+c\sqrt{b}=(a+c)\sqrt{b}$
$2\sqrt{a}-\sqrt[3]{8}-1\sqrt{a}+2\sqrt[3]{27}=\sqrt{a}+4$	2) $\sqrt{a}\times\sqrt{a}=\sqrt{a^2}=a$
$\sqrt{12}+\sqrt{75}=7\sqrt{3}$	3) $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ 4) $\sqrt{a^2\times b}=a\times\sqrt{b}$ 5) $a\sqrt{b}\times c\sqrt{d}=ac\sqrt{bd}$
$(\sqrt{5}-2)(\sqrt{5}+2)=1$ $\frac{2}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{2\sqrt{7}}{7}$	Rationalizing the denominator **What do you think this means?** -changing the denominator from a radical to a rational **How do you think it can be accomplished?** - multiplying the denominator by a root/radical - multiply the numerator to conserve equality

Example:
Rationalize the denominator for the expression.

a) $\frac{5}{\sqrt{8}}$

b) $\frac{2}{\sqrt{5}+3}$ (You need something called conjugate)

Let's work on the examples from Textbook together to get a better understanding!
pg. 164. Example 1, 2, 3, 4, 5, 6

Practice:
pg. 167 # 2, 3, 5, 6 b) d) f), 7 d) f), 12
pg. 170 # 14

Extra practice:

What Do You Call a Group of Factory Foremen Who Sing While Drinking Tab Cola and Eating Crab Apples ?

Simplify each expression below. Assume that all variables represent nonnegative numbers. Find your answer in the corresponding answer column. Write the letter of the exercise in the box that contains the number of the answer.

(E) $\sqrt{5}\cdot\sqrt{3}$	(7) $2x^2\sqrt{6}$	(N) $5\sqrt{2}\cdot 4\sqrt{3}$	(25) $30\sqrt{2}$
(H) $\sqrt{6}\cdot\sqrt{2}$	(2) $10\sqrt{2}$	(B) $-7\sqrt{3}\cdot 2\sqrt{10}$	(11) $5a^2\sqrt{3b}$
(O) $\sqrt{3}\cdot\sqrt{6}$	(3) $12x^5$	(I) $2\sqrt{6}\cdot 5\sqrt{3}$	(8) $-14\sqrt{15}$
(A) $\sqrt{5}\cdot\sqrt{10}$	(9) $\sqrt{15}$	(A) $4\sqrt{10}\ (-3\sqrt{2})$	(4) $36ab\sqrt{6b}$
(R) $\sqrt{27}\cdot\sqrt{3}$	(12) $x\sqrt{6}$	(R) $2\sqrt{8}\cdot\sqrt{18}$	(17) $-24\sqrt{5}$
(H) $\sqrt{10}\cdot\sqrt{20}$	(5) $3\sqrt{2}$	(L) $-10\sqrt{3}\ (-2\sqrt{21})$	(15) $18ab$
(E) $\sqrt{90}\cdot\sqrt{40}$	(1) $3x^2\sqrt{10}$	(M) $-\sqrt{6}\cdot 7\sqrt{10}$	(22) $40a^2b^4\sqrt{6a}$
(A) $\sqrt{2x}\cdot\sqrt{3x}$	(23) $2\sqrt{3}$	(N) $3\sqrt{ab}\cdot 6\sqrt{ab}$	(6) 24
(D) $\sqrt{6x}\cdot\sqrt{2x}$	(26) 9	(P) $\sqrt{2ab^2}\cdot\sqrt{14ab^2}$	(10) $20\sqrt{6}$
(T) $\sqrt{30x^2}\cdot\sqrt{3x^2}$	(21) 60	(T) $-\sqrt{15a^2b}\ (-\sqrt{5a^2})$	(19) $2ab^2\sqrt{7}$
(E) $\sqrt{3x}\cdot\sqrt{8x^3}$	(18) $20x\sqrt{x}$	(O) $\sqrt{8ab^2}\ (-\sqrt{10a^3b^4})$	(13) $-14\sqrt{30}$
(P) $\sqrt{40x^2}\cdot\sqrt{10x}$	(14) $5\sqrt{2}$	(F) $2\sqrt{18a^2b}\cdot 6\sqrt{3b^2}$	(24) $-4a^2b^3\sqrt{5}$
(E) $\sqrt{12x^5}\cdot\sqrt{12x^5}$	(16) $2x\sqrt{3}$	(C) $5\sqrt{2a^3b^8}\cdot 4\sqrt{12a^2}$	(20) $60\sqrt{7}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

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Extra practice continued:

Did you hear about...

A	B	C	D	E	F	G	H
I	J	K	L	M	N	O	P ?

Rationalize the denominator and simplify each expression below. Find your answer in the adjacent answer column and notice the word next to it. Write this word in the box containing the letter of that exercise. Keep working and you will hear about a mistake.

Answers A–H:

$\sqrt{11}$	TO
$\frac{\sqrt{5}}{2}$	WAS
$\frac{\sqrt{2}}{6}$	HUG
$\frac{2\sqrt{10}}{5}$	TRIED
$4\sqrt{5}$	SAD
$\frac{5\sqrt{3}}{3}$	THE
$\frac{3\sqrt{5}}{10}$	BIG
$\frac{\sqrt{6}}{2}$	WHO
$\frac{\sqrt{3}}{2}$	KISS
$\frac{2\sqrt{7}}{7}$	VERY
$7\sqrt{2}$	GUY
$\frac{2\sqrt{6}}{3}$	GIRL

(A) $\frac{5}{\sqrt{3}}$

(B) $\frac{2}{\sqrt{7}}$

(C) $\frac{20}{\sqrt{5}}$

(D) $\frac{14}{\sqrt{2}}$

(E) $\frac{3}{\sqrt{6}}$

(F) $\frac{4}{\sqrt{10}}$

(G) $\frac{11}{\sqrt{11}}$

(H) $\frac{3}{\sqrt{12}}$

(I) $\frac{30}{\sqrt{18}}$

(J) $\frac{8}{\sqrt{20}}$

(K) $\frac{9}{2\sqrt{45}}$

(L) $\frac{\sqrt{7}}{\sqrt{3}}$

(M) $\frac{\sqrt{5}}{\sqrt{10}}$

(N) $\frac{3\sqrt{6}}{\sqrt{2}}$

(O) $\frac{\sqrt{3}}{2\sqrt{6}}$

(P) $\frac{2\sqrt{3}}{\sqrt{15}}$

Answers I–P:

$\frac{3\sqrt{2}}{4}$	BUT
$\frac{\sqrt{2}}{4}$	AND
$\frac{\sqrt{21}}{3}$	IN
$\frac{4\sqrt{5}}{5}$	GIRL
$\frac{6\sqrt{2}}{5}$	LOST
$3\sqrt{3}$	FOG
$\frac{3\sqrt{5}}{10}$	FRIEND
$\frac{\sqrt{2}}{2}$	THE
$5\sqrt{2}$	HIS
$\frac{2\sqrt{2}}{5}$	A
$\frac{2\sqrt{5}}{5}$	MIST
$\frac{9\sqrt{3}}{10}$	TODAY

Extra practice continued:

What Do You Get When You Cross. . .

1. A thief with a cement truck?

5	14	5	12	9	2	13	2	9	11	12	15	7	15	13	5	10

2. A supermarket with a jungle?

11	14	2	11	4	1	8	3	10	15	1	13	6

Express each product below in simplest form. Find your answer and notice the letter next to it. Each time the exercise number appears in the code, write this letter above it. Keep working and you will discover the result of each "double cross."

(1) $(5-\sqrt{2})(5+\sqrt{2})$
(2) $(7+\sqrt{3})(7-\sqrt{3})$
(3) $(\sqrt{10}-6)(\sqrt{10}+6)$
(4) $(\sqrt{10}-\sqrt{6})(\sqrt{10}+\sqrt{6})$
(5) $(\sqrt{2}+8)(\sqrt{2}+3)$

(K) 4
(G) -14
(E) 46
(A) $26+11\sqrt{2}$
(P) $30+9\sqrt{2}$
(O) 23
(T) -26

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(6) $(\sqrt{13}+1)(\sqrt{13}-5)$
(7) $(6-\sqrt{15})(3-\sqrt{15})$
(8) $(9+\sqrt{7})^2$
(9) $3\sqrt{2}(\sqrt{6}+\sqrt{2})$
(10) $2\sqrt{5}(7\sqrt{3}-\sqrt{10})$

(D) $6\sqrt{3}+6$
(S) $8-4\sqrt{13}$
(U) $88+18\sqrt{7}$
(L) $14\sqrt{15}-10\sqrt{2}$
(M) $33-9\sqrt{15}$
(V) $7\sqrt{15}$
(B) $9\sqrt{3}$

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(11) $5\sqrt{3}(2\sqrt{15}+\sqrt{8})$
(12) $3\sqrt{6}(4\sqrt{3}-2\sqrt{15})$
(13) $(8+3\sqrt{5})(1+2\sqrt{5})$
(14) $(2\sqrt{7}+4)(5\sqrt{7}-11)$
(15) $(3\sqrt{10}-5\sqrt{2})(2\sqrt{10}+6\sqrt{2})$

(Y) $8+6\sqrt{7}$
(H) $26-2\sqrt{7}$
(N) $38+19\sqrt{5}$
(C) $30\sqrt{5}+10\sqrt{6}$
(R) $36\sqrt{2}-18\sqrt{10}$
(I) $16\sqrt{5}$
(F) $11\sqrt{5}$

MCR3U – Unit 2 – Review Package

Learning Goals Checklist

- ☐ Verify that two polynomials are equivalent using simplification.
- ☐ Verify that two polynomials are not equivalent using substitution.
- ☐ Add, subtract, multiply, divide and simplify rational expressions using factoring.
- ☐ State the restrictions on the denominator for a rational expression.
- ☐ Perform basic operations with radicals.
- ☐ Rationalize the denominator for an expression involving radicals in the denominator.

Textbook Review

PG. 132 #1-15
PG. 133 #1-8
PG. 167 #1-7
PG. 206 #5-8