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1. The molar solubility is the amount of material that will dissolve in 1 mol of solvent. The product solubility constant, K_{sp}, is the equilibrium constant for the dissolution reaction.

2. A common ion is an ion that is present in two different solutes. Its presence decreases the solubility of the less-soluble substance because it shifts the equilibrium toward the solid.

3. (a) $BaSO_4(s) \rightleftharpoons Ba^{2+}(aq) + SO_4{}^{2-}(aq)$

(b) $K_{sp} = [Ba^{2+}(aq)][SO_4{}^{2-}(aq)]$

4. Both solid silver chloride, $AgCl(s)$, and solid silver bromide, $AgBr(s)$, ionize in a 1:1 ratio, so the compound with the smaller K_{sp} value, $AgBr(s)$, is less soluble in water.

5. Given: solubility of $AgI(s) = 2.14 \times 10^{-7}$ g/100 mL at 25 °C

Required: K_{sp} of $AgI(s)$ at 25 °C

Solution:

Step 1. Write the balanced equation.

$$AgI(s) \rightleftharpoons Ag^+(aq) + I^-(aq)$$

Step 2. Write the solubility product constant equation.

$$K_{sp} = [Ag^+(aq)][I^-(aq)]$$

Step 3. Determine the equilibrium concentrations in mol/L of the dissolved ions. Use an ICE table. Since the concentration of $AgI(s)$ remains constant, it is not included.

	$AgI(s)$	$\rightleftharpoons$	**$Ag^+(aq)$**	+	**$I^-(aq)$**
I	—		0.00		0.00
C	—		$+x$		$+x$
E	—		x		x

Step 4. Calculate all concentrations in mol/L. To convert given mass solubility of solid silver iodide to molar solubility, first calculate the molar mass of silver iodide. Then, use the molar mass to convert mass solubility to molar solubility.

$$\text{molar mass of AgI} = 107.87\ \text{g/mol} + 126.90\ \text{g/mol}$$

$$\text{molar mass of AgI} = 234.77\ \text{g/mol (two extra digits carried)}$$

$$\text{molar solubility of AgI(s)} = \frac{2.14\times10^{-7}\ \cancel{\text{g}}}{100\ \cancel{\text{mL}}} \times \frac{1\ \text{mol}}{234.77\ \cancel{\text{g}}} \times \frac{1000\ \cancel{\text{mL}}}{1\ \text{L}}$$

$$\text{molar solubility of AgI(s)} = 9.115\times10^{-9}\ \text{mol/L (one extra digit carried)}$$

Step 5. Write the K_{sp} equation using the expressions in the ICE table, and then substitute the molar solubility of solid silver iodide for x.

$$\begin{aligned} K_{sp} &= [Ag^+(aq)][I^-(aq)] \\ &= (x)(x) \\ &= (9.115 \times 10^{-9})(9.115 \times 10^{-9}) \\ K_{sp} &= 8.31 \times 10^{-17} \end{aligned}$$

Statement: The solubility product constant of solid silver iodide is 8.31×10^{-17}.

6. Given: (from Table 1 in Appendix B4) K_{sp} of ZnOH(s) at 25 °C = 7.7×10^{-17}
Required: molar solubility of $Zn(OH)_2(s)$ at 25 °C
Solution:
Step 1. Write the balanced chemical equation.

$$Zn(OH)_2(s) \rightleftharpoons Zn^{2+}(aq) + 2\ OH^-(aq)$$

Step 2. Write the solubility product constant equation.

$$K_{sp} = [Zn^{2+}(aq)][OH^-(aq)]^2$$

Step 3. Determine the equilibrium concentrations in mol/L of the dissolved ions. Use an ICE table. Since the concentration of $Zn(OH)_2(s)$ remains constant, it is not included.

	$Zn(OH)_2(s) \rightleftharpoons$	$Zn^{2+}(aq)$ +	$2\ OH^-(aq)$
I	—	0.00	0.00
C	—	$+x$	$+2x$
E	—	x	$2x$

Step 4. Substitute into the equation for the solubility product the value of K_{sp} for $Zn(OH)_2(s)$ and the equilibrium concentration expressions from the ICE table. Solve for x.

$$K_{sp} = [Zn^{2+}(aq)][OH^-(aq)]^2$$

$$K_{sp} = (x)(2x)^2$$

$$7.7 \times 10^{-17} = 4x^3$$

$$x^3 = 1.925 \times 10^{-17}$$

$$x = \sqrt[3]{1.925 \times 10^{-17}}$$

$$x = 2.7 \times 10^{-6}$$

$[Zn^{2+}(aq)]$ = molar solubility of $Zn(OH)_2(s)$
molar solubility of $Zn(OH)_2(s) = 2.7 \times 10^{-6}$ mol/L

Statement: The molar solubility of solid zinc hydroxide is 2.7×10^{-6} mol/L.

7. Given: $[Ca(NO_3)_2(aq)] = 0.01$ mol/L; $[Na_3PO_4(aq)] = 0.025$ mol/L
Required: To predict whether a precipitate will form and identify any precipitate
Solution:
Step 1. Identify all of the ions that will be present in the mixture.
$CaNO_3(aq)$ will contribute aqueous ions of calcium, $Ca^{2+}(aq)$, and nitrate, $NO_3^-(aq)$.
$Na_3PO_4(aq)$ will contribute aqueous ions of sodium, $Na^+(aq)$, and phosphate, $PO_4^{3-}(aq)$.
All four ions will be in the mixture: $Ca^{2+}(aq)$, $NO_3^-(aq)$, $Na^+(aq)$, and $PO_4^{3-}(aq)$.

Step 2. Use the solubility rules in Table 3 on page 465 to predict whether a precipitate may form.
Table 3 indicates that $NaNO_3(s)$ is highly soluble and $Ca_3(PO_4)_2(s)$ has low solubility. Therefore, a precipitate of solid calcium phosphate may form.

Step 3. Determine the concentrations of the ions that may react to form a precipitate.
First, write the balanced dissolution equation for $Ca(NO_3)_2(aq)$.
$Ca(NO_3)_2(s) \rightarrow \mathbf{Ca^{2+}(aq)} + 2\ NO_3^-(aq)$
From the balanced equation, $Ca(NO_3)_2(s)$ and $Ca^{2+}(aq)$ have a 1:1 molar ratio. Therefore, the concentrations of the solute and the ions are equal. Therefore,
$[Ca^{2+}(aq)] = 0.01$ mol/L
Then, write the balanced dissolution equation for $Na_3PO_4(aq)$.
$Na_3PO_4(s) \rightarrow 3\ Na^+(aq) + \mathbf{PO_4^{3-}(aq)}$
From the balanced equation, $Na_3PO_4(s)$ and $PO_4^{3-}(aq)$ have a 1:1 molar ratio. Therefore, the concentrations of the solute and the ions are equal. Therefore,
$[PO_4^{3-}\ (aq)] = 0.025$ mol/L

Step 4. Determine the trial ion product equation of the precipitate that may form.
$Ca_3(PO_4)_2(s)$ may precipitate. First, write a balanced equation for the dissolution equilibrium system of $Ca_3(PO_4)_2(s)$.
$Ca_3(PO_4)_2\ (s) \rightleftharpoons 3\ Ca^{2+}(aq) + 2\ PO_4^{3-}(aq)$
From the balanced equation, the trial ion product equation is
$Q = [Ca^{2+}(aq)]^3[PO_4^{3-}(aq)]^2$

Step 5. Calculate Q by substituting the concentrations of the dissolved ions in the mixture into the trial ion product equation.
$$Q = [Ca^{2+}(aq)]^3[PO_4^{3-}(aq)]^2$$
$$= (0.01)^3(0.025)^2$$
$$Q = 6.2 \times 10^{-10}$$

Step 6. Compare the K_{sp} and Q values of the compound that may precipitate, and then predict whether the position of this dissolution equilibrium system will shift and whether a precipitate will form.
From Table 1 in Appendix B4,
K_{sp} of $Ca_3(PO_4)_2(s) = 2.1 \times 10^{-33}$
$Q = 6.2 \times 10^{-10}$
Q is greater than K_{sp}, so the dissolution equilibrium system will shift to the left and a precipitate of $Ca_3(PO_4)_2(s)$ will form.

Statement: A precipitate of solid calcium phosphate, $Ca_3(PO_4)_2(s)$, will form when the two solutions are mixed.

8. Given: $V_{Mg(NO_3)_2} = 100.0$ mL ; $[Mg(NO_3)_2(aq)] = 4.0 \times 10^{-4}$ mol/L;

$V_{NaOH} = 100.0$ mL ; $[NaOH(aq)] = 2.0 \times 10^{-3}$ mol/L

Required: To predict whether a precipitate will form and identify any precipitate

Solution:

Step 1. The initial $Mg(NO_3)_2(aq)$ solution and initial NaOH(aq) solution will contribute these four ions to the mixture: $Mg^{2+}(aq)$, $NO_3^-(aq)$, $Na^+(aq)$, and $OH^-(aq)$.

Step 2. Use the solubility rules in Table 3 on page 465 to predict whether a precipitate may form.
Table 3 indicates that $NaNO_3$ is highly soluble and $Mg(OH)_2$ has low solubility. Therefore, a precipitate of solid magnesium hydroxide, $Mg(OH)_2(s)$, may form.

Step 3. Determine the concentrations of the ions that may react to form a precipitate.

$Mg(NO_3)_2(aq) \rightarrow \mathbf{Mg^{2+}(aq)} + 2\ NO_3^-(aq)$

The ratio of $[Mg^{2+}(aq)]$ to $[Mg(NO_3)_2(aq)]$ is 1:1.

$NaOH(aq) \rightarrow Na^+(aq) + \mathbf{OH^-(aq)}$

The ratio of $[OH^-(aq)]$ to $[NaOH(aq)]$ is 1:1.

Therefore, $[Mg^{2+}(aq)] = 4.0 \times 10^{-4}$ mol/L and $[OH^-(aq)] = 2.0 \times 10^{-3}$ mol/L. These concentrations are the concentration of $Mg^{2+}(aq)$ ions in the initial $Mg(NO_3)_2(aq)$ solution and the concentration of $OH^-(aq)$ in the initial NaOH(aq) solution *before* these solutions are mixed. When the initial solutions are mixed, the volume of the final mixture, V_f, increases and is the sum of the volumes of the two initial solutions.

$$\begin{aligned} V_\mathrm{f} &= V_{\mathrm{Mg(NO_3)_2}} + V_\mathrm{NaOH} \\ &= 100.0\ \mathrm{mL} + 100.0\ \mathrm{mL} \\ V_\mathrm{f} &= 200.0\ \mathrm{mL} \end{aligned}$$

Use the dilution formula to calculate the concentrations of $Mg^{2+}(aq)$ and $OH^-(aq)$ in the final mixture.

$$c_\mathrm{c}V_\mathrm{c} = c_\mathrm{d}V_\mathrm{d}$$

$$c_\mathrm{d} = \frac{c_\mathrm{c}V_\mathrm{c}}{V_\mathrm{d}}$$

where c_c is the initial, concentrated, ion concentration and c_d is the final, more dilute, ion concentration. V_c and V_d are the corresponding volumes.

Substitute the appropriate values for $Mg^{2+}(aq)$ into the dilution equation.

$$[\mathrm{Mg^{2+}(aq)}]_\mathrm{final} = (4.0 \times 10^{-4}\mathrm{mol/L})\left(\frac{100.0\ \cancel{\mathrm{mL}}}{200.0\ \cancel{\mathrm{mL}}}\right)$$

$$[\mathrm{Mg^{2+}(aq)}]_\mathrm{final} = 2.0 \times 10^{-4}\ \mathrm{mol/L}$$

Then, substitute the appropriate values for $OH^-(aq)$ into the dilution equation.

$$[\mathrm{OH^-(aq)}]_\mathrm{final} = (2.0 \times 10^{-3}\ \mathrm{mol/L})\left(\frac{100.0\ \cancel{\mathrm{mL}}}{200.0\ \cancel{\mathrm{mL}}}\right)$$

$$[\mathrm{OH^-(aq)}]_\mathrm{final} = 1.0 \times 10^{-3}\ \mathrm{mol/L}$$

Step 4. Determine the trial ion product equation of the precipitate that may form.

$$Mg(OH)_2(s) \rightleftharpoons Mg^{2+}(aq) + 2\ OH^-(aq)$$

$$Q = [Mg^{2+}(aq)][OH^-(aq)]^2$$

Step 5. Calculate Q.

$$\begin{aligned} Q &= [Mg^{2+}(aq)][OH^-(aq)]^2 \\ &= (2.0 \times 10^{-4})(1.0 \times 10^{-3})^2 \\ Q &= 2.0 \times 10^{-10} \end{aligned}$$

Step 4. Write the equation for the solubility product, K_{sp}, and substitute in the value of K_{sp} for $AgCrO_4(s)$ from Table 1 in Appendix B4, and the equilibrium concentration expressions from the ICE table. Solve for *x*.

$$K_{sp} = [Ag^+(aq)]^2[CrO_4^{2-}(aq)]$$

$$1.1 \times 10^{-12} = (2x)^2(x + 0.10)$$

$$1.1 \times 10^{-12} = 4x^2(x + 0.10)$$

$$x^2 = \frac{1.1 \times 10^{-12}}{4(x + 0.10)}$$

$$x = \sqrt{\frac{1.1 \times 10^{-12}}{4(x + 0.10)}}$$

For chromate ions, $AgCrO_4(s)$ solubility is low, so the amount of $CrO_4^{2-}(aq)$ ions that the $AgCrO_4(s)$ will add to the solution will be very small compared to the 0.10 mol/L provided by the $Na_2CrO_4(aq)$ solution. Therefore, make the simplifying assumption that $x + 0.10 \approx 0.10$.

$$x \approx \sqrt{\frac{1.1 \times 10^{-12}}{4(0.10)}}$$

$$x \approx 1.7 \times 10^{-6}$$

Statement: The molar solubility of silver chromate in a 0.10 mol/L aqueous solution of sodium chromate is 1.7×10^{-6} mol/L.

10. The solubility product is the product of the ion concentrations of a saturated solution. The trial ion product is the product of ion concentrations of a real or proposed solution.

11. (a) Answers may vary. Sample answer: Solutions that could be used to precipitate out the phosphate from the water are solutions of the salts of a non-Group 1 metal. Two examples of such salts are $CaCl_2(aq)$ or $Sr(NO_3)_2(aq)$.

(b) As predicted by Le Châtelier's principle, the addition of the cation causes a stress to the equilibrium system that is relieved by shifting the equilibrium toward the solid phosphate salt.

12. Answers may vary. Sample answer: The formation of stalactites and stalagmites in limestone caves is an example of solubility equilibrium in the natural environment. Calcium carbonate from limestone dissolves in ground water and forms an equilibrium between the dissociated and solid calcium carbonate. There is also an equilibrium between carbon dioxide, carbonate ions, and hydrogen carbonate ions. As carbon dioxide leaves the solution into the air, this equilibrium shifts in favour of increased carbonate ion concentration. When the carbonate ion concentration increases, equilibrium with calcium carbonate shifts in favour of solid calcium carbonate, which precipitates to form a solid. Solid calcium carbonate deposits hanging down from the tops of caves are stalactites. Where the solution drips to the ground and the precipitate forms there the deposits are called stalagmites.