



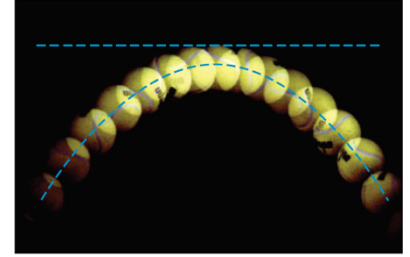
Unit 1: Kinematics

Lesson 1.3 (Chapter 1.5): Projectile Motion

A **projectile** is an object that only moves under the influence of gravity.

A **trajectory** is the path a projectile takes through the air.

The vertical and horizontal components of motion are independent.

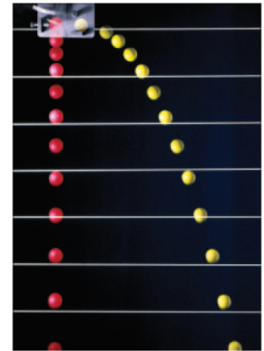


The path of a projectile follows the curve of a parabola.

Suppose you drop a soccer ball from the roof while your friend stands next to you and kicks another soccer ball horizontally at the same instant under the same height. Will they both land at the same time? Why?

The answer is yes! Because:

- 1) The horizontal motion of a projectile is constant. No acting of any force. Thus no acceleration.
- 2) The vertical acceleration of both soccer balls is $g = 9.8m/s^2$.
- 3) Independent horizontal and vertical motions share the same time.



Questions?

- 1) What angle should you fire for maximum **time of flight**?
- 2) What angle should you fire for maximum **range**?
- 3) What angle should you fire for maximum **height**?
- 4) Can two firing angles result in the same horizontal range?

Example: Solving Projectile Motion problems with an initial vertical velocity

A golfer hits a golf ball with an initial velocity of 25 m/s at an angle of 30.0° above the horizontal. The golfer is at an initial height of 14 m above the point where the ball lands.

- a) Calculate the maximum height of the ball.
- b) Determine the ball's velocity on landing.



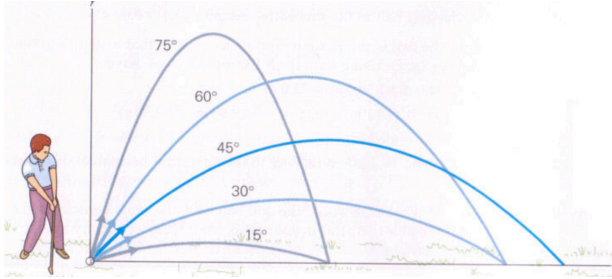
The Time & Range Equation suppose the projectile were launched and landed at the same height (i.e., $\Delta d_y = 0$):

$$\text{Time: } \Delta t = \frac{2v_i \sin \theta}{g}$$

$$\text{Range: } \Delta d_x = \frac{v_i^2}{g} \sin 2\theta$$

Pg. 41.

Hence: The largest value the range can have occurs when $\theta =$ _____



If $\Delta d_y = h$, then the angle makes maximal range is: $\theta = \frac{1}{2} \sec^{-1} \left(1 + \frac{R_0}{h} \right)$, where $R_0 = \Delta d_x = \frac{v_i^2}{g} \sin 2\theta$ when $\theta = 45^\circ$
Hence, when shooting is above ground, i.e., $h \uparrow$, then $\theta \downarrow$. Normally between $30^\circ - 35^\circ$.

Practice:

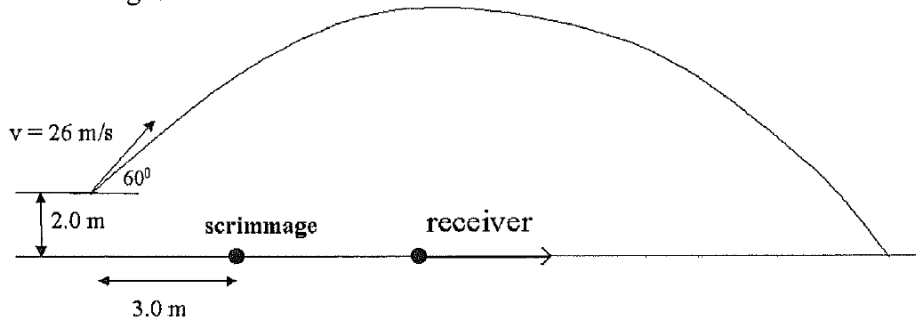
1. Suppose you kick a soccer ball at 28 m/s toward the goal at a launch angle of 21° . a) How long does the soccer ball stay in the air? b) Determine the distance the soccer ball would need to cover to score a goal (the range).

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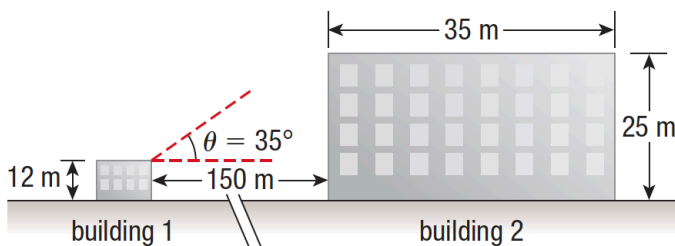
2. Projectile motion with 2 objects

A football quarterback attempts a pass to one of the receivers. As the ball is snapped, the receiver leaves the line of scrimmage and runs directly down field. The quarterback releases the ball 2.0 s later and from a position 3.0 m behind the line of scrimmage. He throws the ball with a speed of 26 m/s at an elevation of 60° above the horizontal. The receiver makes a diving reception, catching the ball just as it reaches the ground. See the diagram below.



- (a) What is the time of flight of the football?
- (b) What is the average speed of the receiver?

7. A projectile launcher launches a snowball at 45 m/s from the top of building 1 in **Figure 7**. Does the snowball land on top of building 2? Support your answer with calculations. T/I





SPH4U - Projectile Motion

1. Explain why an airplane moving through the air is not an example of projectile motion.
5. Two identical bullets are at the same height. One bullet is fired horizontally from a rifle at a velocity of 1000 m/s over level ground. The other bullet, released at the same instant, falls straight down. How long does it take each bullet to reach the ground? Explain your answer.
8. A baseball is thrown straight up in the air. Describe the baseball's velocity and acceleration at each of the following points:
 - a) half-way up.
 - b) at its maximum height.
 - c) half-way down.
10. A baseball pitcher throws a ball horizontally under negligible air resistance. The ball falls 83 cm in travelling 18.4 m to the home plate. Determine the ball's initial horizontal speed.
30. Blarney, the orange dinosaur, throws a Nerf™ ball horizontally out of an open window with a velocity of 3.0 m/s. If the window is 10 m above the ground, how far away from the building must Blarney's friend stand to catch the ball at ground level?
31. A rock thrown horizontally from the top of a water tower lands 20.0 m from the base of the tower. If the rock was initially thrown at a velocity of 10.0 m/s,
 - a) how high is the water tower?
 - b) What is the final velocity of the rock?
32. A bag of mail is catapulted from the top of a building 200 m above the ground with a velocity of 20 m/s at an angle of 15° above the horizontal. If the mail is to land on the roof of another building 100 m away, how tall is the second building?
33. A tourist taking the train from Toronto, Ontario to Montreal, Quebec accidentally drops a cup of coffee from a height of 1.3 m.

The train is traveling at 180 km/h.

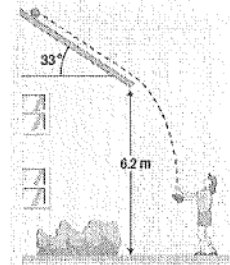
 - a) How long does it take the cup of coffee to hit the floor?
 - b) Where does the cup land relative to the tourist?
 - c) How much closer to Montreal is the cup when it strikes the floor compared to when it was dropped?
34. Bounder of Adventure is trying to cross a piranha-infested pool of water in his Humvee. He races up a ramp inclined at 20° to the horizontal at a speed of 30 m/s.

There is an identical ramp on the other side of the pool. What is the maximum width of the pool that Bounder of Adventure can successfully cross?

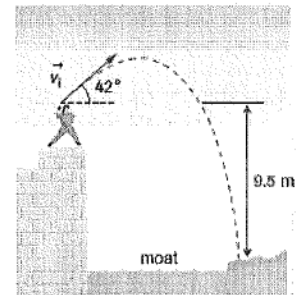
Fig.2.58



35. A soccer ball is kicked from the ground at an angle Θ above the horizontal. Show that the equation $h = 0.25R \tan \Theta$ represents the maximum height of the ball, where h is the height and R is the range.
36. A baseball player makes perfect contact with the ball, striking it 45° above the horizontal at a point 1.3 m above the ground. His homerun hit just clears the 3.0-m wall 130 m from home plate. With what velocity did the baseball player strike the ball?
40. A child throws a ball onto the roof of a house, then catches it with a baseball glove 1.0 m above the ground. The ball leaves the roof with a speed of 3.2 m/s.
 - (a) For how long is the ball airborne after leaving the roof?
 - (b) What is the horizontal distance from the glove to the edge of the roof?
 - (c) What is the velocity of the ball just before it lands in the glove?



45. A medieval prince trapped in a castle wraps a message around a rock and throws it from the top of the castle wall with an initial velocity of 12 m/s [42° above the horizontal]. The rock lands just on the far side of the castle's moat, at a level 9.5 m below the initial level. Determine the rock's
 - (a) time of flight
 - (b) width of the moat
 - (c) velocity at impact



NUMERICAL ANSWERS:

10. 45 m/s
30. 4.2 m
31. a) 19.6 m
b) 22 m/s, 63 degree below hor
32. 95 m
33. a) 0.52 s
b) At tourist's feet
c) 26 m
34. 59 m
36. 36 m/s, 45° above horizontal
40. (a) 0.87s (b) 2.3 m (c) 11m [75° below Hoz]
45. (a) 2.4 s (b) 22 m (c) 18 m/s [60° below the horizontal]