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Unit 2: Work and Momentum

Lesson 2.5: Elastic collisions and inelastic collisions

Due to the change of velocities, collision can be classified into two different types in according to whether the total kinetic energy of the entire system is conserved or not.

https://www.youtube.com/watch?v=M2xnGcaaAi4

Elastic Collisions: is a collision in which **momentum and kinetic energy are conserved**, where the objects involved remain separate after the collision and friction, sounds and other external forces are negligible.

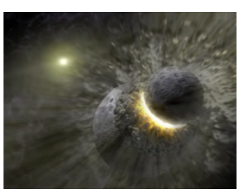
We can consider an ideal rubber ball, the rubber balls are compressed when they get collided momentarily, and such compression stores energy in the ball just as the energy stored in a compressed spring. Once they bounce off each other, potential energy is turned back into kinetic energy.

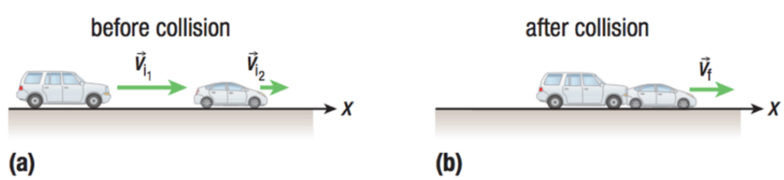
Inelastic collisions: collision in which **momentum is conserved, but kinetic energy is not conserved** due to the loss of thermal energy or sound energy.

Inelastic collisions can involve a ball composed of soft clay whose nature and texture does not allow the return of shape deformation. In other words, energy is absorbed, causing the kinetic energy after the collision to be less than the kinetic energy before the collision.

Perfect inelastic collisions: is one in which the two objects in a collision stick together to become one mass after the collision so that the objects have the same final velocity.

Examples: Celestial bodies (two asteroid) collide to fuse to one larger body; Car accident





In summary:

- In an elastic collision, both momentum and kinetic energy are ________________.
- In an inelastic collision, __________________ is conserved, but _______________________ is not conserved.

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$
$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

Example 1: Suppose you have two balls with different masses involved in a perfectly elastic collision. Ball 1, with mass 0.26 kg travelling at a velocity 1.3 m/s [right], collides head-on with stationary ball 2, which has a mass of 0.15 kg. Determine the final velocities of both balls after the collision.

Example 3: A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child "sticks" on the rope swing and swings forward. How high do the child and swing rise?

Practice: Textbook pg236. #1, 2 & pg238. #1 – 3 & pg239. #1 – 8

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Head-on collisions & Conservation of Mechanical Energy

Perfectly Elastic Head-on Collisions in One Dimension

In a one-dimensional **head-on elastic collision**, two objects approach each other from opposite directions and collide. In such collisions, both momentum and kinetic energy are conserved. You can derive expressions for the final velocities of two objects in a head-on collision in terms of the initial velocities and the objects' masses.

Suppose an object of mass m_1 travels with initial velocity v_{i_1} and collides head-on with an object of mass m_2 travelling at velocity v_{i_2}. If we assume a one-dimensional collision, we can omit the vector notation for velocities, and instead use positive and negative values to identify motion in one direction or the opposite direction. We begin the analysis with the conservation of momentum:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2} \qquad \text{(Equation 1)}$$

Rewrite Equation 1 by bringing all the terms with m_1 to one side and all the terms with m_2 to the other side, and common factoring the m coefficients:

$$m_1v_{i_1} - m_1v_{f_1} = m_2v_{f_2} - m_2v_{i_2}$$

$$m_1(v_{i_1} - v_{f_1}) = m_2(v_{f_2} - v_{i_2}) \qquad \text{(Equation 2)}$$

Since this is an elastic collision, conservation of total kinetic energy can be applied:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

Multiply both sides of the equation by 2 to clear the fractions:

$$\cancel{2}\left(\frac{1}{\cancel{2}}m_1v_{i_1}^2 + \frac{1}{\cancel{2}}m_2v_{i_2}^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}m_1v_{f_1}^2 + \frac{1}{\cancel{2}}m_2v_{f_2}^2\right)$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

Collect m_1 terms on the left side and m_2 terms on the right side and divide out the common factors.

$$m_1v_{i_1}^2 - m_1v_{f_1}^2 = m_2v_{f_2}^2 - m_2v_{i_2}^2$$

$$m_1(v_{i_1}^2 - v_{f_1}^2) = m_2(v_{f_2}^2 - v_{i_2}^2)$$

Factor both sides using the difference of squares:

$$m_1(v_{i_1} - v_{f_1})(v_{i_1} + v_{f_1}) = m_2(v_{f_2} - v_{i_2})(v_{f_2} + v_{i_2}) \qquad \text{(Equation 3)}$$

Divide Equation 3 by Equation 2:

$$\frac{\cancel{m_1(v_{i_1} - v_{f_1})}(v_{i_1} + v_{f_1})}{\cancel{m_1(v_{i_1} - v_{f_1})}} = \frac{\cancel{m_2(v_{f_2} - v_{i_2})}(v_{f_2} + v_{i_2})}{\cancel{m_2(v_{f_2} - v_{i_2})}}$$

$$v_{i_1} + v_{f_1} = v_{f_2} + v_{i_2} \qquad \text{(Equation 4)}$$

Rearranging Equation 4 to isolate v_{f_2} on the left gives

$$v_{f_2} = v_{i_1} + v_{f_1} - v_{i_2} \qquad \text{(Equation 5)}$$

Substitute Equation 5 into Equation 1 to express v_{f_1} in terms of the masses and their initial speeds:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2(v_{i_1} + v_{f_1} - v_{i_2})$$

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{i_1} + m_2v_{f_1} - m_2v_{i_2}$$

Collect the v_{f_1} terms on the right side of the equation and terms involving initial velocities on the left side, then collect like terms and divide out common m coefficients:

$$m_1v_{i_1} - m_2v_{i_1} + m_2v_{i_2} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_1}$$

$$(m_1 - m_2)v_{i_1} + 2m_2v_{i_2} = (m_1 + m_2)v_{f_1}$$

Divide both sides by $m_1 + m_2$ to isolate v_{f_1}:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

This equation expresses the final velocity of the first object in terms of the masses and initial velocities of the two objects. Similarly, we can rearrange Equation 4 to isolate v_{f_1} on the left:

$$v_{f_1} = v_{f_2} + v_{i_2} - v_{i_1} \qquad \text{(Equation 6)}$$

To derive a similar equation for v_{f_2}, follow the above steps for v_{f_1}, starting with substituting Equation 6 into Equation 1, and ending by dividing both sides by $m_1 + m_2$ and isolating v_{f_2} on the left side:

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

This equation expresses the final velocity of the second object in terms of the masses and initial velocities of the two objects. Note that the equation for $\vec{v}_{f_2}$ is the same as the equation for $\vec{v}_{f_1}$ if you interchange all the 1 and 2 subscripts. It is important to note that these equations hold true only for perfectly elastic collisions in one dimension.

In some cases, one of the objects is initially at rest. For instance, if v_2 is initially zero, the equations above simplify to

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Special Cases

Using these new equations for head-on elastic collisions in one dimension, special cases of collisions, such as objects of equal mass, produce some interesting results.

CASE 1: OBJECTS HAVE THE SAME MASS

The first case we consider is when the objects that are colliding have the same mass, so let $m_1 = m_2 = m$.

$$\vec{v}_{f_1} = \left(\frac{m - m}{m + m}\right)\vec{v}_{i_1} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_2}$$

$$= \left(\frac{0}{2m}\right)\vec{v}_{i_1} + \left(\frac{2m}{2m}\right)\vec{v}_{i_2}$$

$$= \left(\frac{2m}{2m}\right)\vec{v}_{i_2}$$

$$\vec{v}_{f_1} = \vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m - m}{m + m}\right)\vec{v}_{i_2} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_1}$$

$$= \left(\frac{0}{2m}\right)\vec{v}_{i_2} + \left(\frac{2m}{2m}\right)\vec{v}_{i_1}$$

$$= \left(\frac{2m}{2m}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_2} = \vec{v}_{i_1}$$

In other words, when two objects with the same mass undergo a head-on elastic collision in one dimension, they exchange velocities almost as if they pass through each other.

CASE 2: A LIGHTER OBJECT COLLIDING WITH A MUCH HEAVIER, STATIONARY OBJECT

Our second case deals with situations in which the mass of one of the objects is much greater than the mass of the other object, and the heavier object is stationary. For example, if object 2 is stationary and has a much greater mass, then since m_2 is much greater than m_1, you can consider m_1 to be approximately zero, or negligible. So

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$\approx \left(\frac{0 - m_2}{0 + m_2}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_1} \approx -\vec{v}_{i_1}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$\approx \left(\frac{2(0)}{0 + m_2}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_2} \approx 0$$

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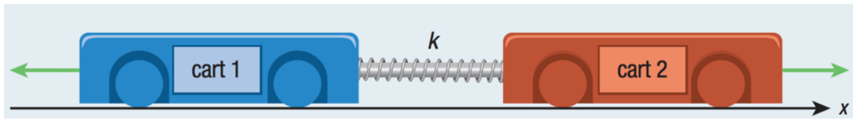
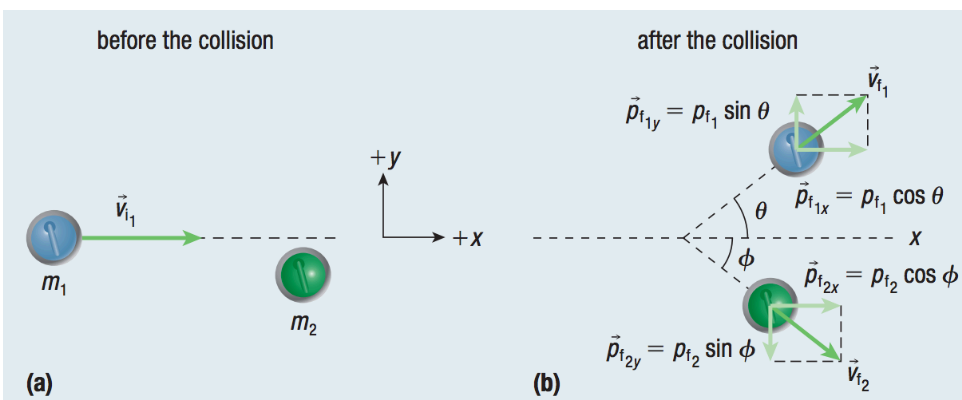
Example 3: Dynamics cart I has a mass of 1.8 kg and is moving with a velocity of 4.0 m/s [right] along a frictionless track. Dynamics cart 2 has a mass of 2.2 kg and is moving at 6.0 m/s [left]. The carts collide in a head-on elastic collision cushioned by a spring with spring constant $k = 8.0 \times 10^4 N/m$.



a) Determine the compression of the spring, in centimetres, during the collision when cart 2 is moving at 4.0 m/s [left].
b) Calculate the maximum compression of the spring, in centimetres.

Practice: Textbook pg247. #1, 2 & pg248. #1 – 6

Example 5: Analysis of a Glancing collision in two dimension
In a game of curling, a collision occurs between two stones of equal mass. The object stone is initially at rest. After the collision, the stone that is thrown has a speed of 0.56 m/s in some direction, represented by θ in the diagram below. The object stone acquires a velocity $\overrightarrow{v_{f2}} = 0.42 m/s$ at an angle of $\varphi = 30.0°$ from the original direction of motion of the thrown stone. Determine the initial velocity of the thrown stone.



Practice: Textbook pg253. #1 – 4, 8