Unit 3: Gravitational, electric, and magnetic fields
Lesson 3.2: Change in Potential Energy - AP Physics

Recall:

- $F_g = \frac{GmM}{r^2}$
- $g = \frac{GM}{r^2}$
- $E_g = -\frac{GmM}{r}$
- In an orbit: $E_k = -\frac{1}{2}E_g = \frac{1}{2}\frac{GmM}{r}$
- In an orbit: $E_T = \frac{1}{2}E_g$

A change in potential energy can be found by using:

- $\Delta E_g = E_{gf} - E_{gi} = -\frac{GmM}{r_f} - \left(-\frac{GmM}{r_i}\right) = \frac{GmM}{r_i} - \frac{GmM}{r_f} = GmM(\frac{1}{r_i} - \frac{1}{r_f})$

Case 1: Send a satellite from Earth surface (with radius of R) to an orbit (certain distance r away)

https://www.youtube.com/watch?v=9rEPjhFAOZM&list=PL58F689B9528DEBC4&index=3

Most of time, scientists need to know how much work is required to send a satellite or spacecraft to a certain orbit, so they will have to know how much propellant (i.e., fuel) is needed and prepare for that accordingly.

$$E_{ki} + E_{gi} = E_{Tf}$$

$$W + (-\frac{GmM}{R}) = -\frac{1}{2}\frac{GmM}{r}$$

So $W =$

Example 1: Wishing to launch a 60 kg toy spacecraft from the surface of Earth to an orbit $7 \times 10^6 m$ far away, find work to go into orbit and find velocity to go into orbit. Given the radius of Earth is $6.4 \times 10^6 m$ and mass of the Earth is $6.0 \times 10^{24} kg$.

Case 2: Send a satellite from Earth surface to certain distance away but not in an orbit ($\boldsymbol{V_f = 0}$)

https://www.youtube.com/watch?v=jKWltfG-VjU&list=PL58F689B9528DEBC4&index=1

$\boldsymbol{E_{ki} + E_{gi} = E_{kf} + E_{gf}}$

$\boldsymbol{E_{ki} + E_{gi} = \ 0 \ + E_{gf}}$

$\boldsymbol{W = E_{gf} - E_{gi}}$ This is how much work to input to send the satellite away from the Earth surface.

Example 2: Wishing to launch a 50 kg toy spacecraft from the surface of Earth to $7 \times 10^6 m$ far away (NOT in orbit), find work input and velocity. Given the radius of Earth is $6.4 \times 10^6 m$ and mass of the Earth is $6.0 \times 10^{24} kg$.

Case 3: Send a satellite from one orbital, $\boldsymbol{r_1}$, to another, $\boldsymbol{r_2}$, where , $\boldsymbol{r_1 < r_2}$

Use energy conservation:

$\boldsymbol{E_{ki} + E_{gi} + W = E_{kf} + E_{gf}}$ or $\boldsymbol{W = E_{Tf} - E_{Ti}}$

$$\boldsymbol{W = \frac{1}{2} GmM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)}$$

Example 3: a) What is the change in gravitational potential energy of a 4500 kg Earth satellite move from an orbital radius of $1.8 \times 10^7 m$ to an orbital radius of $4.2 \times 10^7 m$?

b) If the 4500 kg satellite now moves from $1.8 \times 10^7 m$ back to $4.2 \times 10^7 m$, how much work is required?

Case 4: Send a satellite from Earth surface to infinite far ($V_f = 0$)

https://www.youtube.com/watch?v=_Ms9A9fM0ew&list=PL58F689B9528DEBC4&index=2

Binding Energy: The minimum kinetic energy an object requires in order to break free.

$$E_B = -E_{Ti} = -E_{gi} = \frac{GmM}{R}$$

Escape velocity: What goes up must come down, unless we throw it really, really hard. So escape velocity is the minimum speed an object requires in order to break **free** from Earth's pull. It should stand to reason that if an object is going to be completely free from the Earth gravitational pull, we need to supply it with enough ____________________ to match its ______________________ at initial.

$$E_{ki} + E_{gi} = 0 + 0$$

$$E_{ki} = -E_{gi} = \frac{GmM}{R} = \frac{1}{2}mv^2$$

$$V_{escape} = \sqrt{\frac{2GM}{R}}$$

Example 4: At what speed do you need to throw a 1.0 kg rock in order for it to leave the Earth's gravitational pull? Does the mass of the rock matter?

Practice:

1. The international Space Station drops a 250 kg waste shuttle from an altitude of $3.50 \times 10^5 m$. At what speed would it impact Earth if there were no air friction? (Assume it starts at rest).
2. What is the change of total energy if a spacecraft change orbit from $3.50 \times 10^5 m$ to $2.50 \times 10^5 m$? Is total energy increased or decreased?
3. An 200 kg unknown alien is planning to become completely unbounded from the Mars, determine binding energy.

Orbiting velocity (6.2)

Orbiting velocity is the velocity that a satellite could have travelled under certain distance away from the central planet.

$F_c = F_g$ provides $\frac{mv^2}{r} = \frac{GmM}{r^2}$ $\rightarrow$

$\boldsymbol{v = \sqrt{\frac{GM}{r}}}$ which is the speed of the satellite that orbiting around the central planet.

The speed of satellite is only associated with orbital radius and parent body mass.

Example 5: Determine the speeds of Venus as it orbits the Sun. The Sun's mass is $1.99 \times 10^{30} kg$. And Venus has an orbital radius of $1.08 \times 10^{11} m$.

Example 6: Calculate the orbital radius of a satellite in geosynchronous orbit.

Example 5: Determine the speed of a satellite, in kilometres per hour, that is in a geosynchronous orbit about Earth. (If we only know period and mass of Earth)

Example 6: Saturn makes one complete orbit of the Sun every 29 Earth years with a speed of 9.69 km/s. Calculate the radius of the orbit of Saturn. Assume a circular orbit and Mass of Sun is $1.989 \times 10^{30} kg$.

Practice: Textbook pg303. #6, 7

The end of Chapter 6.