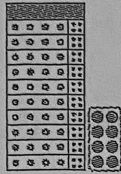


- 1: The board game Mastermind® uses a four-colour code that can be made up of any of six colours: red, white, green, blue, black, and yellow. One player creates a code and the second player tries to solve it in as few attempts as possible.



How many different codes are possible if

- i) four different colours are used for the code?
- ii) three different colours are used (with one colour repeated)?
- iii) two different colours are used?

- 2: a) Determine all of the prime factors of 360.  
b) How many factors are there of 360?

- 3: At a party, each person shakes hands with every other person. If there are a total of 66 handshakes, how many people are at the party?

- 4: How many ways can you distribute ten apples among five people if each person gets at least one apple?

- 5: You have three quarters, two loonies, and four toonies. How many different groups of coins can be made if
- a) there are no restrictions?
  - b) there must be at least three coins in each group?

- 6: A new pizza parlour offers five toppings and three pizza sizes to choose from.

- a) How many different pizzas can be made from the menu?
- b) The owner wants to name the restaurant Pizza 2000 to indicate all the different types of pizza that can be made. She increases the number of options to include three different cheeses, of which you can choose one, two, or three. If the owner wants to offer more than 2000 possible pizzas, how many topping choices must she make available?

- 7: MegaBurger is a new restaurant. You can create your own custom hamburger using the guidelines below. How many different types of hamburger can you make at MegaBurger?

Required Elements	Optional Elements
Bun (7 types)	Condiments (10 types)
Patty (12 types)	Cheeses (8 types)
	Vegetables (6 types)
	Spicy bits (4 types)

Answer:

1. (i)  ${}^6P_4 = 360$

(ii)  ${}^6C_3 \times \frac{4!}{2!} = 240$

(iii) (3) + (1)  $({}^6C_2 \times \frac{4!}{3!}) \times 2 = 120$

(2) + (2)  $({}^6C_2 \times \frac{4!}{2! \cdot 2!}) = 90$

$\therefore 90 + 120 = 210$

2. a)  $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

b) (1) 1 is divisor/factor

(2) take one 3

(3) take two  $2 \times 2, 2 \times 3, 2 \times 5, 3 \times 3, 3 \times 5$

$(3+1)(2+1)(1+1)$   
 $= 4 \cdot 3 \cdot 2 = 24$

(4) take three  $222, 223, 225, 233, 235, 335$

(5) take four  $2223, 2225, 2235, 2233, 2235$

(6) take five  $22233, 22235, 22335$

(7) take six all.

$\therefore$  Total  $24$

3.  ${}^nC_2 = 66 \quad \frac{n!}{(n-2)!2!} = 66 \quad \rightarrow n = 12$  people

4. A A A A A A A A A A  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

$\therefore {}^9C_4 = 126$  ways

To divide them into 5 piles you need 4 dividers.

5 a)  $(3+1)(2+1)(5+1) - 1 = 59$

b) All - one coin - two coins

$= 59 - 3 - (3 + 3) = 50$

same two different  
\$1 \$1 1 25  
\$2 \$2 1 2  
\$75 \$75 2 25

3 - \$75  
2 - loonies 1  
4 - toonies 2

$$6. a) 3 \times 2^5 = 96.$$

$$b) \frac{2000}{3} = 667.$$

$$2^9 < 667 < 2^{10}.$$

∴ 10 different toppings → 7 different cheese = 7.

$$\frac{\text{Size} \times \text{Cheese} \times \text{topping}}{3 \times (2^3 - 1) \times 2^t} \geq 2000$$

$$t \geq 7$$

$$7: 7 \times 12 \times (11+1)(8+1)(6+1)(4+1) =$$

$$7 \times 12 \times (2^{10+8+6+4}) = 22548578304$$