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Unit 3: Probability

Lesson 3.2: Odds

- Odds in favor of an event's occurring are given by the ratio of the probability that the event will occur to the probability that it will not occur.

  Odds in favor of $\mathrm{A} = \frac{P(A)}{P(A')} = \frac{n(A)}{n(A')}$

- Odds against $\mathrm{A} = \frac{P(A')}{P(A)}$, this is the reciprocal of odds in favor of A.

- If the odds in favor of A are $\frac{h}{k}$, then $\mathrm{P(A)} = \frac{h}{h+k}$

Example 1: A messy drawer contains three red socks, five white socks, and four black socks. What are the odds in favor of randomly drawing a red sock?

Example 2: If the chance of a snowstorm in Windsor, Ontario, in January is estimated at 0.4, what are the odds against Windsor's having a snowstorm next January? Is a January snowstorm more likely than not?

Notes:

If odds in favor of A is great than 1 (numerator is bigger than denominator), the event will be likely to happen;

If odds in favor of A is less than 1 (numerator is smaller than denominator), the event will be not likely to happen;

If odds against A is greater than 1, the event will not likely to happen;

If odds against A is less than 1, the event will likely to happen.

Example 3: A university professor, in an effort to promote good attendance habits, states that the odds of passing her course are 8 to 1 when a student misses fewer than five classes. What is the probability that a student with good attendance will pass?

This tells us odds in favor of passing the course with good attendance is 8:1 (or as fraction $\frac{8}{1}$)

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Practise

A

1. Suppose the odds in favour of good weather tomorrow are 3:2.

a) What are the odds against good weather tomorrow?

b) What is the probability of good weather tomorrow?

2. The odds against the Toronto Argonauts winning the Grey Cup are estimated at 19:1. What is the probability that the Argos will win the cup?

3. Determine the odds in favour of rolling each of the following sums with a standard pair of dice.

a) 12 **b)** 5 or less

c) a prime number **d)** 1

4. Calculate the odds in favour of each event.

a) New Year's Day falling on a Friday

b) tossing three tails with three coins

c) not tossing exactly two heads with three coins

d) randomly drawing a black 6 from a complete deck of 52 cards

e) a random number from 1 to 9 inclusive being even

Apply, Solve, Communicate

B

5. Greta's T-shirt drawer contains three tank tops, six V-neck T-shirts, and two sleeveless shirts. If she randomly draws a shirt from the drawer, what are the odds that she will

a) draw a V-neck T-shirt?

b) not draw a tank top?

6. **Application** If the odds in favour of Boris beating Elena in a chess game are 5 to 4, what is the probability that Elena will win an upset victory in a best-of-five chess tournament?

7. Chapter Problem **a)** Based on the randomly tagged sample, what are the odds in favour of a captured deer being a cross-hatched buck?

b) What are the odds against capturing a doe?

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site and follow the links for more information about Canadian wildlife.

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8. The odds against A, by definition, are equivalent to the odds in favour of A'. Use this definition to show that the odds against A are equal to the reciprocal of the odds in favour of A.

9. **Application** Suppose the odds of the Toronto Maple Leafs winning the Stanley Cup are 1:5, while the odds of the Montréal Canadiens winning the Stanley Cup are 2:13. What are the odds in favour of either Toronto or Montréal winning the Stanley Cup?

10. What are the odds against drawing

a) a face card from a standard deck?

b) two face cards?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application

11. Mike has a loaded (or unfair) six-sided die. He rolls the die 200 times and determines the following probabilities for each score:

$P(1) = 0.11$
$P(2) = 0.02$
$P(3) = 0.18$
$P(4) = 0.21$
$P(5) = 0.40$

a) What is $P(6)$?

b) Mike claims that the odds in favour of tossing a prime number with this die are the same as with a fair die. Do you agree with his claim?

c) Using Mike's die, devise a game with odds in Mike's favour that an unsuspecting person would be tempted to play. Use probabilities to show that the game is in Mike's favour. Explain why a person who does not realize that the die is loaded might be tempted by this game.

12. George estimates that there is a 30% chance of rain the next day if he waters the lawn, a 40% chance if he washes the car, and a 50% chance if he plans a trip to the beach. Assuming George's estimates are accurate, what are the odds

a) in favour of rain tomorrow if he waters the lawn?

b) in favour of rain tomorrow if he washes the car?

c) against rain tomorrow if he plans a trip to the beach?

C

13. **Communication** A volleyball coach claims that at the next game, the odds of her team winning are 3:1, the odds against losing are 5:1, and the odds against a tie are 7:1. Are these odds possible? Explain your reasoning.

14. **Inquiry/Problem Solving** Aki is a participant on a trivia-based game show. He has an equal likelihood on any given trial of being asked a question from one of six categories: Hollywood, Strange Places, Number Fun, Who?, Having a Ball, and Write On! Aki feels that he has a 50/50 chance of getting Having a Ball or Strange Places questions correct, but thinks he has a 90% probability of getting any of the other questions right. If Aki has to get two of three questions correct, what are his odds of winning?

15. **Inquiry/Problem Solving** Use logic and mathematical reasoning to show that if the odds in favour of A are given by $\frac{h}{k}$, then $P(A) = \frac{h}{h+k}$. Support your reasoning with an example.

Unit 3: Probability

Lesson 3.3: Independent Events and Dependent Events

Part I: Independent Events

If the occurrence of one event has no effect on the occurrence of another, the events are ____________.

Product Rule (Fundamental Counting Principle) for independent Events
If A and B are independent events, then the probability of both occurring is given by: $$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

<u>Example 1</u>:

a) A coin is tossed 4 times turning up heads each time. What is the probability the fifth trial will be heads?

Explain: Each coin toss is __________________ of the others. Although we may think "heads has to come up sometime", there is still only a half chance on each independent toss. The coin has no memory of the previous four tosses. Therefore, the 5^{th} toss has a ________ probability of getting heads.

b) Find the probability of tossing five heads in a row.

$P(H \cap H \cap H \cap H \cap H) = P(H) \times P(H) \times P(H) \times P(H) \times P(H) =$ ______________________________

Hence, there is a ________ chance of getting 5 heads in a row.

c) A coin is flipped while a die is rolled. What is the probability that you will flip a tail and roll a four?

$P(Tail \cap 4) = P(Tail) \times P(4) =$ ___________________________

Hence, the probability of tossing tails and rolling a 4 is _________

<u>Example 2:</u> There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that

a) both tests give the correct result.
b) neither test gives the correct result
c) at least one of the tests gives the correct result

<u>Example 3:</u> Pinder has examinations coming up in data management and biology. He estimates that his odds in favour of passing the data-management examination are 17:3 and his odds against passing the biology examination are 3:7. Assume these to be independent events.

a) What is the probability that Pinder will pass both exams?
b) What are the odds in favour of Pinder failing both exams?

Part II: Dependent events

If the occurrence of one event influences the probability of another event, the events are ________________.

When this happens, you can still multiply their probabilities of each event to find the likelihood of A and B (i.e., $P(A \cap B)$). However, you must use the **<u>conditional probability</u>** for the second event.

The conditional probability of B, $P(B|A)$, is the probability that B occurs, given that A has already occurred. The sample space for the second event is reduced by the first event.

Product Rule (Fundamental Counting Principle) for Dependent Events
The probability of dependent events occurring is $P(A\ and\ B) = P(A) \times P(B|A)$
By rearranging the formula, conditional probability is $P(B|A) = \frac{P(A\ and\ B)}{P(A)}$

<u>Example 4</u>:

- Part 1 - Suppose you have a bag containing 2 black marbles and 3 red marbles. You reach into the bag, select a marble, see what color it is and replace it in the bag (Event #1). Then you repeat this process (Event #2). What is the probability of picking a red marble both times?

  <u>Solution</u>: Since the first marble was replaced back in the bag before the second marble was drawn, the probability of the second drawing is independent of the probability of the first drawing. These are referred to as independent events --- in other words, the outcome of one event does not affect the outcome of the other event.

- Part 2 - Suppose you have a bag containing 2 black marbles and 3 red marbles. You reach into the bag, select a marble, see what color it is but **DO NOT** replace it in the bag (Event #1). Then you reach in and select another marble (Event #2). What is the probability of picking a red marble both time?

  <u>Solution:</u> The probability of picking a red marble the first time (Event #1) is the same as it was in part 1), which is 3 out of 5, or ($\frac{3}{5}$). However, since the first marble was not replaced back in the bag, the probability

of picking a red marble the second time is dependent on the outcome of the first drawing. Suppose we did pick a red marble and did not put it back in the bag. Now there are only 2 red marbles and 2 black marbles in the bag, and the probability of picking a red marble the second time (Event #2) is 2 out of 4. Therefore, in this case, Event #1 and Event #2 occurring simultaneously will have a probability of the product of the two dependent event probabilities.

- Hence: **An independent event occurs with replacement. A dependent event occurs without replacement.**

Independent: when the outcome of one event does not change the probability of the other.

Dependent: the probability of the second outcome depends on the results of the first.

Example 5: Tell whether the events are independent or dependent.

a) You select a card from a deck of 52 cards, replace that card, and select another card.

b) Ms. Ella chooses students at random to present their projects. She chooses you, and then another student from the remaining students.

c) There are 10 winning tickets in a collection of 500 tickets. You select a ticket, put it aside, and select another ticket.

Example 6: Lars is offering juice samples at a shopping mall. The experimental probability of a randomly chosen shopper accepting a sample is 15%. The conditional probability of a customer purchasing juice given that they tried a sample is 20%. No one purchases juice without trying a sample. If Lars offers 500 people juice samples, how many sales will he make?

Let A be "try sample", B be "buy juice", and A and B will be ____.

Namely, $P(A) =$ ________ and _________ $= 0.2$

Hence: $P(A\ and\ B) = P(A) \times P(B|A) =$

Example 7: Rocco and Biff are two koala bears participating in a series of animal behavior tests. They each have 10 mins to solve a maze. Rocco has an 85% probability of succeeding if he can smell the treat. He can smell the treat 60% of the time. Biff has a 70% chance of smelling the treat, but when he does, he can solve the maze only 75% of the time. Neither bear will try to solve the maze unless he smells the treats. Determine which koala bear is more likely to enjoy a tasty treat on any given trial.

Example 8: Grace is a kicker on her rugby team. She estimates that her chances of scoring on a penalty kick during a game are 75% when there is no wind, but only 60% on a windy day. If the weather forecast gives a 55% chance of windy weather this afternoon, what is the probability of Grace scoring on a penalty kick in this afternoon's game?

Example 9: The table shows the status of 200 registered college students. A student is randomly selected.

	Part time	Full time
Female	80	40
Male	60	20

a) What is the probability that the selected student is female?

b) What is the probability that a selected female student is a full time student?
P (B|A) is the probability of a selected female is full time student;
P (A and B) is the probability of a student is both a female and a full time student;
P (A) is the probability of a selected student is a female.

Practise

A

1. Classify each of the following as independent or dependent events.

	First Event	Second Event
a)	Attending a rock concert on Tuesday night	Passing a final examination the following Wednesday morning
b)	Eating chocolate	Winning at checkers
c)	Having blue eyes	Having poor hearing
d)	Attending an employee training session	Improving personal productivity
e)	Graduating from university	Running a marathon
f)	Going to a mall	Purchasing a new shirt

2. Amitesh estimates that he has a 70% chance of making the basketball team and a 20% chance of having failed his last geometry quiz. He defines a "really bad day" as one in which he gets cut from the team and fails his quiz. Assuming that Amitesh will receive both pieces of news tomorrow, how likely is it that he will have a really bad day?

3. In the popular dice game Yahtzee®, a Yahtzee occurs when five identical numbers turn up on a set of five standard dice. What is the probability of rolling a Yahtzee on one roll of the five dice?

Apply, Solve, Communicate

B

4. There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that

a) both tests give the correct result

b) neither test gives the correct result

c) at least one of the tests gives the correct result

5. a) Rocco and Biff are two koala bears participating in a series of animal behaviour tests. They each have 10 min to solve a maze. Rocco has an 85% probability of succeeding if he can smell the eucalyptus treat at the other end. He can smell the treat 60% of the time. Biff has a 70% chance of smelling the treat, but when he does, he can solve the maze only 75% of the time. Neither bear will try to solve the maze unless he smells the eucalyptus. Determine which koala bear is more likely to enjoy a tasty treat on any given trial.

b) Communication Explain how you arrived at your conclusion.

6. Shy Tenzin's friends assure him that if he asks Mikala out on a date, there is an 85% chance that she will say yes. If there is a 60% chance that Tenzin will summon the courage to ask Mikala out to the dance next week, what are the odds that they will be seen at the dance together?

7. When Ume's hockey team uses a "rocket launch" breakout, she has a 55% likelihood of receiving a cross-ice pass while at full speed. When she receives such a pass, the probability of getting her slapshot away is $\frac{1}{3}$. Ume's slapshot scores 22% of the time. What is the probability of Ume scoring with her slapshot when her team tries a rocket launch?

8. Inquiry/Problem Solving Show that if A and B are dependent events, then the conditional probability $P(A \mid B)$ is given by

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}.$$

9. A consultant's study found Megatran's call centre had a 5% chance of transferring a call about schedules to the lost articles department by mistake. The same study shows that, 1% of the time, customers calling for schedules have to wait on hold, only to discover that they have been mistakenly transferred to the lost articles department. What are the chances that a customer transferred to lost articles will be put on hold?

10. Pinder has examinations coming up in data management and biology. He estimates that his odds in favour of passing the data-management examination are 17:3 and his odds against passing the biology examination are 3:7. Assume these to be independent events.

a) What is the probability that Pinder will pass both exams?

b) What are the odds in favour of Pinder failing both exams?

c) What factors could make these two events dependent?

11. Inquiry/Problem Solving How likely is it for a group of five friends to have the same birth month? State any assumptions you make for your calculation.

12. Determine the probability that a captured deer has the bald patch condition.

Chapter Problem

13. Communication Five different CD-ROM games, Garble, Trapster, Zoom!, Bungie, and Blast 'Em, are offered as a promotion by SugarRush cereals. One game is randomly included with each box of cereal.

a) Determine the probability of getting all 5 games if 12 boxes are purchased.

b) Explain the steps in your solution.

c) Discuss any assumptions that you make in your analysis.

14. Application A critical circuit in a communication network relies on a set of eight identical relays. If any one of the relays fails, it will disrupt the entire network. The design engineer must ensure a 90% probability that the network will not fail over a five-year period. What is the maximum tolerable probability of failure for each relay?

C

15. a) Show that if a coin is tossed n times, the probability of tossing n heads is given by $P(A) = \left(\frac{1}{2}\right)^n$.

b) What is the probability of getting at least one tail in seven tosses?

16. What is the probability of not throwing 7 or doubles for six consecutive throws with a pair of dice?

17. Laurie, an avid golfer, gives herself a 70% chance of breaking par (scoring less than 72 on a round of 18 holes) if the weather is calm, but only a 15% chance of breaking par on windy days. The weather forecast gives a 40% probability of high winds tomorrow. What is the likelihood that Laurie will break par tomorrow, assuming that she plays one round of golf?

18. Application The Tigers are leading the Storm one game to none in a best-of-five playoff series. After a playoff win, the probability of the Tigers winning the next game is 60%, while after a loss, their probability of winning the next game drops by 5%. The first team to win three games takes the series. Assume there are no ties. What is the probability of the Storm coming back to win the series?